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Causal Analysis of Kagawa Prefecture Economic Data [1]

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Abstract

This paper aims to apply the Wald test of causality to the analysis of regional economic data. For this purpose we first show the approach which can test the causal characteristics expressed in terms of the causal measures of the one-way effect for cointegrated vector time series. Based on an error correction model, we give a characterization of the economic causal structure of Kagawa Prefecture.

Keywords: Causality, Cointegration, Regional economy, VAR Model.

1 Introduction

Since Granger (1963) introduced the concept of non-causality, which is a statistically testable criterion defined in terms of predictability, there have been many contributions to causal analyses between pairs of time series. The earlier representative studies are the Granger test of zero restriction of specific coefficients of a stationary autoregressive representation, and the Sims test of the zero restriction of some coefficients in moving-average representation of stationary bivariate processes [see Granger (1969), Sims (1972), Hosoya (1977)].

As regards testing Granger's non-causality in levels of a nonstationary vector autoregressive (VAR) system, Sims, Stock and Watson (1990) dealt with trivariate VAR systems, to conclude that the Wald test statistic has a limiting χ^2 distribution if the time series are cointegrated and otherwise that it has a nonstandard limiting distribution. Lütkepohl and Reimers (1992), using the Wald test for Granger's non-causality

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in bivariate cointegrated finite order AR process, investigated the short and long-term interest rates in the U.S., whereas Toda and Phillips (1993) extended the results of Sims, Stock and Watson (1990). So far, the interest of the econometric literature seems mostly concerned with the Granger non-causality test.

For the purpose of quantitative characterization of the feedback relationship between two multivariate time series, Hosoya (1991, 1997) introduced three causal measures summarizing the interdependency between a pair of nondeterministic stationary or nonstationary reproducible processes. In Yao and Hosoya (2000), based on overall as well as frequency-wise causal measures, for the purposes of testing causal relations in cointegrated processes and constructing their confidence-sets, the Wald statistic was proposed. In contrast to the conventional tests of Granger's non-causality, which amount to testing the hypothesis of zero restriction of a certain set of autoregressive coefficients, the one-way effect approach enables us to examine a variety of causal characteristics between time-series; it can test not only Granger's non-causality by means of testing the nullity of the overall measure of the one-way effect (OMO), but also the strength of the one-way effect. Moreover, by means of the integral of the frequency-wise measure of the one-way effect (FMO) on specific frequency bands, the long-run and short-run causal relationships can also be tested. In view of these points, the approach extends conventional causal test theories.

As for the causal analysis of the Japanese macroeconomy in Granger's sense, we can find some remarkable discussions in Tsukuda and Miyakoshi (1998) and Morimune and Zhao (1997). In view of the one-way effect causal measures, Yao and Hosoya (2000) gives a detailed discussions on the Japanese macroeconomy. In this paper we apply the Wald test approach to monthly economic data used in the business cycle analysis of the economy of Kagawa Prefecture from the past twelve years. This is the first study in which the cointegration and one-way effect methods are applied to the investigation of economic data at the prefectural level. The empirical analyses show that the Employment Index of Regular Workers is very active and gives one-way effects to other two indices. The one-way effect of Active Opening Ratio to Clearings is observed.

This paper is organized as follows: Section 2 summarizes the *OMO* and the *FMO* for the nonstationary processes generated by an error correction model (ECM). Based on the ECM, computational procedures of the Wald statistic for testing the *OMO* are shown in detail. Section 3 presents a preliminary data analysis of Kagawa Prefecture's macroeconomic time-series in order to identify pertinent ECMs for the causal analysis. We then apply Johansen's likelihood ratio test to cointegration rank identification and apply extensively the Hosking statistic and the Doornik-Hansen statistic for testing serial uncorrelation and Gaussianity of the residuals. Section 4 deals with empirical causal analysis of the selected data of the Kagawa economy from the past twelve years. The estimates of the *FMO* and *OMO* as well as the Wald statistics for bivariate models are shown in figures. The identified cointegration rank is also listed in the corresponding figures. For the cases where causality was statistically significant, the confidence intervals of the true *OMO* are given in the corresponding figures. Section 5 concludes the paper.

2 The ECM Model and Causality Test

In this section we consider the Wald tests for testing hypotheses on the measures of one-way effect based on the ECM given by (2.1) below, providing the computational procedure and also applying the Wald statistics to the construction of confidence-sets of measures that whould have been rejected under Granger's noncausality. For mathematical details see Hosoya (1991, 1997), Yao and Hosoya (2000), or Yao (2002).

Let $\{Z(t)\} = \{X(t)^*, Y(t)^*\}^*$ be generated by a cointegrated p-vector AR model which is represented in the error-correction form

$$\Delta Z(t) = \alpha\beta^* Z(t-1) + \sum_{j=1}^{a-1} \Gamma(j)\Delta Z(t-j) + \mu + \varepsilon(t), \tag{2.1}$$

where α and β are $p \times r$ matrices ($r \leq p$), and μ is a constant p-vector, $\{\varepsilon(t)\}$ is a Gaussian white noise process with mean 0 with positive definite and non-degenerate variance-covariance matrix Σ.

Let θ be a $(r \cdot p) \times 1$ vector consisting of the elements of β such $\theta = vec\beta^*$. Here and after, the *vec* operator transforms a matrix into a vector by stacking the columns of the matrix one underneath the other. Denoting $n_\psi = p \cdot (r + p \cdot (a-1)) + p \cdot (p+1)/2$, let ψ be the $n_\psi \times 1$ vector which consists of the elements of α and $\Gamma(j)$ $(j = 1, \cdots, a-1)$ and the elements in the lower triangular part of Σ; namely $\psi = vec(vec(\alpha, \Gamma)^*, v(\Sigma))$, where $\Gamma = \{\Gamma(1), \cdots, \Gamma(a-1)\}$ and $v(\Sigma)$ denotes the $(p \cdot (p+1)/2) \times 1$ vector obtained from $vec\Sigma$ by eliminating all supradiagnoal elements of Σ.

The spectral density matrix f and its canonical factor Λ derived for the joint process $\{Z(t)\}$ generated by (2.1) are given respectively by

$$f(\lambda|\theta, \psi) = \frac{1}{2\pi}\Lambda(e^{-i\lambda}|\theta, \psi)\Lambda(e^{-i\lambda}|\theta, \psi)^*, \tag{2.2}$$

and

$$\Lambda(e^{-i\lambda}|\theta, \psi) = C(e^{-i\lambda}|\theta, \psi)\Sigma^{1/2},$$

where $C(e^{-i\lambda}|\theta, \psi)$ is the adjoint matrix of the complex-valued polynomial matrix

$$I_p - e^{-i\lambda}(I_p + \alpha\beta^*) - \sum_{j=1}^{a-1}\Gamma(j)(e^{-ij\lambda} - e^{-i(j+1)\lambda}).$$

It is important to note here that the Granger causality does not involve such deterministic components as the dummy variables and the intercept which appear in model (2.1). This is because there is no one-way effect but only a reciprocal one between deterministic components.

For $i, j = 1, 2$, denote f_{ij} and Σ_{ij} the $p_i \times p_j$ partitioned matrices of $f(\lambda)$ and Σ, respectively. Then the Frequency Measure of One-way effect (FMO) of $\{Y(t)\}$ to $\{X(t)\}$ at frequency λ can be defined by

$$M_{Y \to X}(\lambda|\theta, \psi) = \log[det f_{11}(\lambda)/det\{f_{11}(\lambda) - \tilde{f}_{12}(\lambda)\tilde{f}_{22}^{-1}(\lambda)\tilde{f}_{21}(\lambda)\}], \tag{2.3}$$

where $\tilde{f}_{11}(\lambda) = f_{11}(\lambda)$, $\tilde{f}_{21}(\lambda) = \{-\Sigma_{21}\Sigma_{11}^{-1}, I_{p_2}\}\Lambda(0)\Lambda(e^{-i\lambda})^{-1}f_{\cdot 1}(\lambda)$, $f_{\cdot 1}(\lambda)$ is the $p \times p_1$ matrix which consists of the first p_1 columns of $f(\lambda)$, $\tilde{f}_{22}(\lambda) = \frac{1}{2\pi}\{\Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}\}$. The OMO of $\{Y(t)\}$ to $\{X(t)\}$ can be defined by

$$G(\theta, \psi) = \frac{1}{2\pi}\int_{-\pi}^{\pi} M_{Y \to X}(\lambda|\theta, \psi)d\lambda. \tag{2.4}$$

Note that in these instances, $G(\theta, \psi)$ is differentiable functions with respect to (θ, ψ).

Remark 2.1. *The existence of the Nyquist frequency seems often ignored in exclusively time-domain oriented causal analyses. The observable highest frequency is $\lambda = \pi$, which corresponds with two periods ($t = 2\pi/\pi = 2$); namely, half a year for quarterly data. The economic implication is that we cannot observe the one-way effect in a period shorter than half a year for quarterly data.*

Johansen (1991) showed that, if (θ, ψ) is the true value and $(\hat{\theta}, \hat{\psi})$ is the ML estimate, $T(\hat{\theta}-\theta)$ tends to have a mixed multivariate normal distribution and $\sqrt{T}(\hat{\psi}-\psi)$ tends to have a multivariate normal distribution as $T \to \infty$, whence $G(\hat{\theta}, \hat{\psi})$ is a $\sqrt{T}$ consistent estimate of $G(\theta, \psi)$. By the stochastic expansion, we have

$$\sqrt{T}\{G(\hat{\theta}, \hat{\psi}) - G(\theta, \psi)\} = (D_\psi G)^* \sqrt{T}(\hat{\psi} - \psi) + o_p(1),$$

where $D_\psi G$ is a n_ψ-dimensional vector of the gradient of $G(\theta, \psi)$. It follows that $\sqrt{T}\{G(\hat{\theta}, \hat{\psi}) - G(\theta, \psi)\}$ is asymptotically normally distributed with mean 0 and variance

$$H(\theta, \psi) = D_\psi G(\theta, \psi)^* \Psi(\theta, \psi) D_\psi G(\theta, \psi), \tag{2.5}$$

where $\Psi(\theta, \psi)$ is the asymptotic variance-covariance matrix of $\sqrt{T}(\hat{\psi} - \psi)$. Note that the first-order asymptotic distribution of $G(\hat{\theta}, \hat{\psi})$ is completely determined by $\hat{\psi}$ and the nonstandard limiting distribution of $\hat{\theta}$ is not involved, the sampling error of $\hat{\theta}$ being negligible in comparison with that of $\hat{\psi}$. Consequently, the test for $G(\theta, \psi)$ and the confidence-set construction can be conducted based on the Wald statistic

$$\mathrm{W} \equiv T\{G(\hat{\theta}, \hat{\psi}) - G(\theta_0, \psi_0)\}^2 / H(\hat{\theta}, \hat{\psi}), \tag{2.6}$$

which is asymptotically distributed as χ^2 distribution with one degree of freedom if (θ_0, ψ_0) is the true value.

As regards evaluation of $D_\psi G$ at $\hat{\theta}, \hat{\psi}$, the numerical differentiation would be practical in view of the complexity of the exact analytic expression. Specifically, the gradient of $G(\theta, \psi)$

$$D_\psi G = (\frac{\partial G}{\partial \psi_1}, \cdots, \frac{\partial G}{\partial \psi_{n_\psi}})^*$$

is evaluated by

$$\frac{\partial G}{\partial \psi_i} \approx \{G(\hat{\theta}, \hat{\psi} + h_i) - G(\hat{\theta}, \hat{\psi} - h_i)\}/(2h), \tag{2.7}$$

for sufficiently small positive h where h_i is the $n_\psi \times 1$ vector with the i-th element h and all the other elements zero; namely, $h_i = (0, \cdots, h, \cdots, 0)^*, i = 1, 2, \cdots, n_\psi$.

The computation of $\Psi(\theta, \psi)$ in (2.5) can be conducted as follows. We set $\psi^{(1)} = vec\{\alpha, \Gamma\}$, $\psi^{(2)} = \mu$ and $\psi^{(3)} = \nu(\Sigma)$, and also we set $\psi^{(12)} = vec(\psi^{(1)}, \psi^{(2)})$. Then the log-likelihood function of the parameter $\psi^{(12)}$ and $\psi^{(3)}$ based on observations $Z(1), \cdots, Z(T)$ can be given as

$$l_T(\psi^{(12)}, \psi^{(3)}|Z) = -\frac{T}{2}(p \log 2\pi + \log det\Sigma) - \frac{1}{2} tr\Sigma^{-1} V_T,$$

where

$$V_T = \sum_{t=1}^{T} V(t)V(t)^*,$$

and

$$V(t) = \Delta Z(t) - \alpha\beta^* Z(t-1) - \sum_{j=1}^{a-1} \Gamma(j)\Delta Z(t-j) - \mu.$$

Let D be the p^2 by $p(p+1)/2$ duplication matrix and let D^+ be the Moore-Penrose inverse of matrix D [see Magnus and Neudecker (1988), p49]. Denote by $\hat{\psi}^{(12)}$ and $\hat{\psi}^{(3)}$ the ML estimators of $\psi^{(12)}$ and $\psi^{(3)}$ respectively, $\otimes$ the kronecker product, 1_p the p-dimensional vector with all the elements of 1, then the asymptotic variance-covariance matrix of $\sqrt{T}\{\hat{\psi}^{(12)} - \psi^{(12)}\}$ and $\sqrt{T}(\hat{\psi}^{(3)} - \psi^{(3)})$ is equal to

$$\begin{pmatrix} \Sigma \otimes Q^{-1} & 0 \\ 0 & 2D^+(\Sigma \otimes \Sigma)D^{+*} \end{pmatrix}, \tag{2.8}$$

where $Q = \lim_{T\to\infty}(1/T)\sum_{t=1}^{T} S(t)S(t)^*$,

$$S(t) = vec(\beta^* Z(t-1), \Delta Z(t-1), \cdots, \Delta Z(t-a-1), 1_p)$$

[see Magnus and Neudecker (1995), p321]. The asymptotic covariance of $\sqrt{T}(\hat{\psi}^{(1)} - \psi^{(1)})$, which is denoted by $\Psi_{\psi^{(1)}\psi^{(1)}}$ is then constructed from $\Sigma \otimes Q^{-1}$ by eliminating the rows and columns corresponding to $\sqrt{T}(\hat{\psi}^{(2)} - \psi^{(2)})$. In fact we can write the symmetric $(p \cdot (r + p \cdot (a-1)) + p)$ dimensional matrix $\Sigma \otimes Q^{-1}$ into $p \times p$ partitioned matrix in the form of

$$\begin{pmatrix} \sigma_{11}Q^{-1} & \sigma_{12}Q^{-1} & \cdots & \sigma_{1p}Q^{-1} \\ \sigma_{21}Q^{-1} & \sigma_{22}Q^{-1} & \cdots & \sigma_{2p}Q^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{p1}Q^{-1} & \sigma_{p2}Q^{-1} & \cdots & \sigma_{pp}Q^{-1} \end{pmatrix},$$

where all of the submatrices $\sigma_{ij}Q^{-1}(i,j=1,\cdots,p)$ are $(r+p\cdot(a-1)+1)$ dimensional squared matrix. The covariance matrix $\Psi_{\psi^{(1)}\psi^{(1)}}$ is constructed by eliminating all the last column and the last row of the submatrices $\sigma_{ij}Q^{-1}, i,j=1,\cdots,p$.

As for the estimation of Σ and Q in (2.8), we set

$$\hat{\Sigma} = (1/T)\sum_{t=1}^{T}(\hat{V}(t)\hat{V}(t)^*), \tag{2.9}$$

$$\hat{Q} = (1/T)\sum_{t=1}^{T}\hat{S}(t)\hat{S}(t)^*, \tag{2.10}$$

where

$$\hat{V}(t) = \Delta Z(t) - \hat{\alpha}\hat{\beta}^* Z(t-1) - \sum_{j=1}^{a-1}\hat{\Gamma}(j)\Delta Z(t-j) - \hat{\mu},$$

and

$$\hat{S}(t) = vec(\hat{\beta}^* Z(t-1), \Delta Z(t-1), \cdots, \Delta Z(t-a-1), I_p).$$

In view of the consistency of $\hat{\psi}$ and $\hat{\theta}$, if $\hat{\Psi}_{\psi^{(1)}\psi^{(1)}}$ denotes the variance-covariance matrix of $\sqrt{T}(\hat{\psi}^{(1)} - \psi^{(1)})$ evaluated at $(\hat{\theta}, \hat{\psi})$, then

$$\Psi(\theta,\psi) = \begin{pmatrix} \hat{\Psi}_{\psi^{(1)}\psi^{(1)}} & 0 \\ 0 & 2D^+(\hat{\Sigma}\otimes\hat{\Sigma})D^{+*} \end{pmatrix} + o_p(1). \tag{2.11}$$

Therefore we can use the first part at the right-hand side as a consistent estimate of $\Psi(\theta,\psi)$.

By (2.8) and (2.11), we then get a covariance-matrix estimate $\hat{H} = H(\hat{\theta},\hat{\psi})$. For the purpose of testing the null hypothesis $G(\theta,\psi) = G(\theta_0,\psi_0)$, we evaluate the test statistic W defined by (2.6). In order to the test no-causality in Granger's sense, we set the null hypothesis as $G(\theta,\psi) = 0$ and the test statistic is given by

$$\mathrm{W} = T\{G(\hat{\theta},\hat{\psi})\}^2/H(\hat{\theta},\hat{\psi}). \tag{2.12}$$

If $W \geq \chi^2_\alpha(1)$, for $\chi^2_\alpha(1)$ the upper α quantile of the χ^2 distribution with one degree of freedom, we may reject the null hypothesis of non-causality from Y to X. On the other hand, in view of (2.6), the $(1-\alpha)$ confidence interval of the causal measure $G(\theta,\psi)$ is provided by

$$(\, G(\hat{\theta},\hat{\psi}) - H_\alpha,\; G(\hat{\theta},\hat{\psi}) + H_\alpha \,), \tag{2.13}$$

where $H_\alpha = \sqrt{(1/T)H(\hat{\theta},\hat{\psi})\chi^2_\alpha(1)}$.

3 Preliminary Analysis

The Wald test of causal measures is applied in this section to macroeconomic data that are used in the business cycle analysis of Kagawa Prefecture. The data used are the monthly observations during the period of January 1987 through October 1998. The selected indices are New Dwelling Construction Starts (NDCS), Active Opening Ratio (AOR), Index of Industrial Production (IIP), Building Construction Starts (floor space) (BCS, meters square), Employment Index of Regular Workers (EIRW) and Clearings (Cls, one million yen). The NDCS, AOR, BCS and Cls are seasonally adjusted by the X-11 method, and except for the AOR, the other three indices are given in logarithmic scale. The IIP is seasonally adjusted by the MITI(III) method. These data are presented by the Statistical Section of the General Affairs Department of Kagawa Prefecture. Figure 1 depicts the original data in levels and in differences. All the time series appear, to a reasonable extent, non-stationary with stationary differences.

In the following studies of the causal relations among the data used in the business cycle analysis of Kagawa prefecture, we apply the common lag-length $a = 6$. In order to avoid the lag-length playing a part in differentiation of the configuration, we do not use information criteria which are better suited for identification of individual models. As is seen below, the uncorrelation and the Gaussianity hypotheses seem mostly supported by the residuals derived by fitting the lag-length $a = 6$. The fitted model we used is the cointegrated p-dimensional AR(6) in ECM form represented by

$$\Delta Z(t) = \Pi Z(t-1) + \sum_{k=1}^{5} \Gamma(k)\Delta Z(t-k) + \mu + \varepsilon(t), \tag{3.1}$$

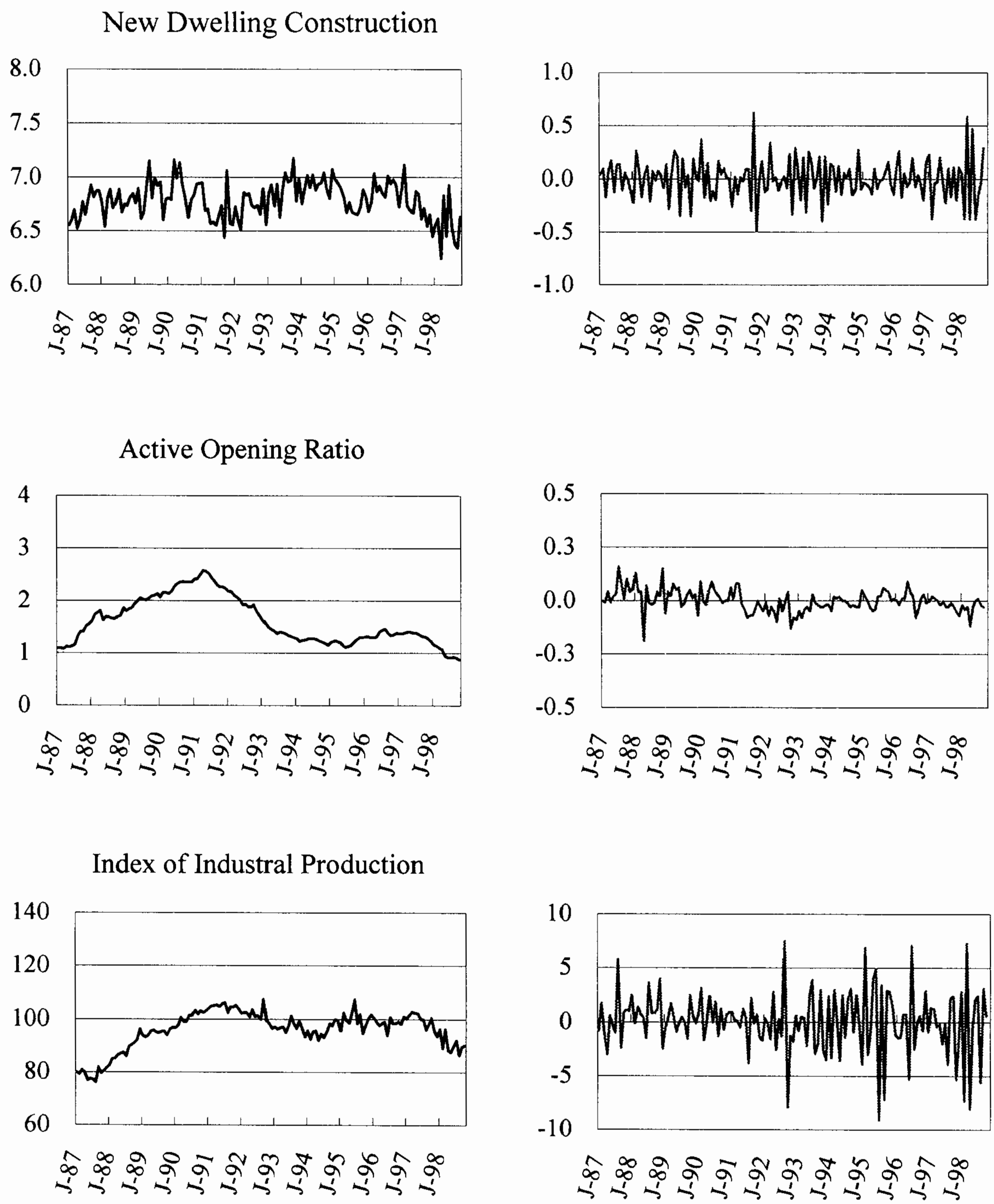
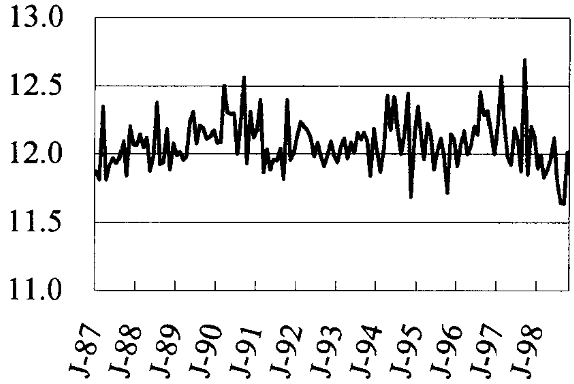
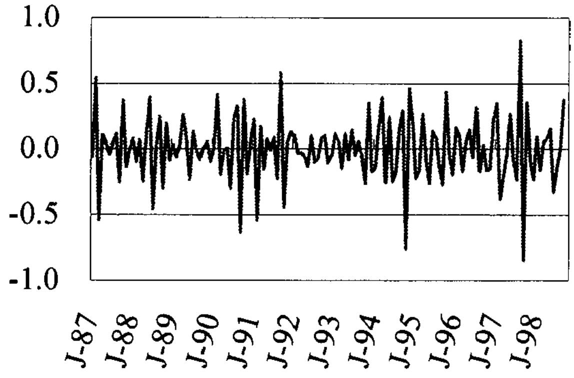


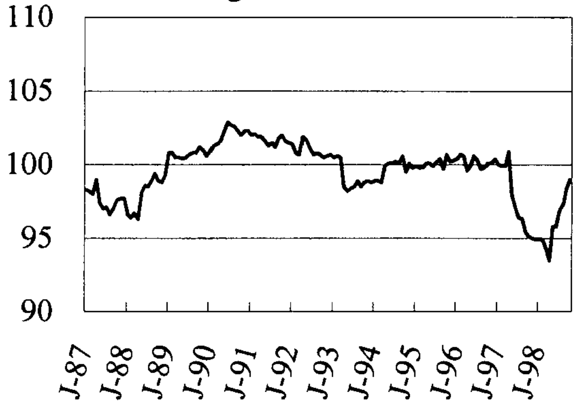
Figure 1. Kagawa Economic Data in Levels and Differences

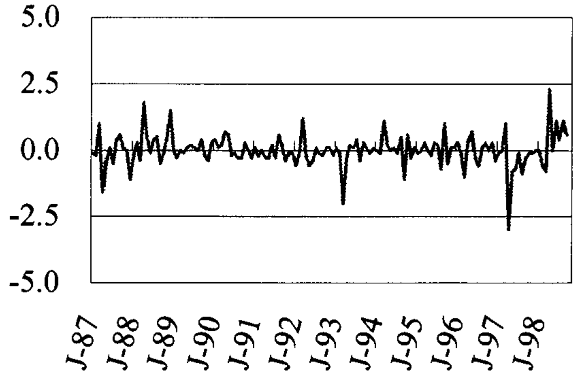
The Building Construction Starts, Floor Space



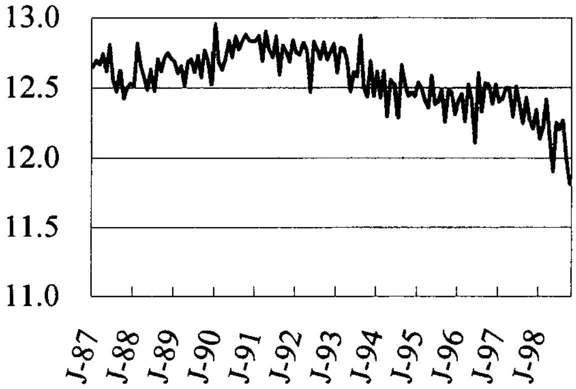


Employment Index of Regular Workers





Clearings



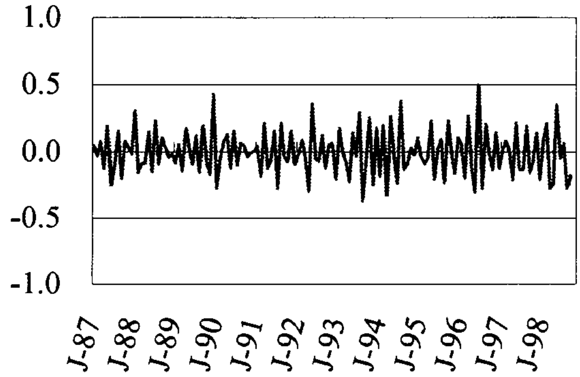


Figure 1. Continue

where $\varepsilon(t)$'s $(t = 1, \cdots, T)$ are Gaussian white noise with mean 0 and variance-covariance matrix Σ. The first 6 observations of $Z(t)$ are kept for initial values.

The hypothesis of independent r cointegration vectors is

$$H(r) : \Pi = \alpha\beta^*, \tag{3.2}$$

where α, β are $p \times r$ matrices $(r \leq p)$ such that $\text{rank}(\Pi) = r$. If $r = 0$, (3.1) reduces to a full-rank unit root process. If $r = p$, then Π is full rank and the process $Z(t)$ is stationary. Denote by $R_0(t)$ and $R_1(t)$ the residuals obtained by regressing $\Delta Z(t)$ and $Z(t-1)$ on $\Delta Z(t-1), \cdots, \Delta Z(t-k+1), 1_p$ respectively. Define a $p \times p$ matrix S_{ij} by

$$S_{ij} = T^{-1} \sum_{t=1}^{T} R_i(t) R_j(t)^*, \quad (i, j = 0, 1). \tag{3.3}$$

Under the hypothesis (3.2), the ML estimator of Π is found by the following procedure [see Johansen (1995), Theorem 6.1]:

(1) First solve the equation

$$|\lambda S_{11} - S_{10} S_{00}^{-1} S_{01}| = 0, \tag{3.4}$$

which produces the decreasing sequence of eigenvalues $1 > \hat{\lambda}_1 > \cdots > \hat{\lambda}_p > 0$ and the matrix constituted by the corresponding eigenvectors $\hat{V} = (\hat{V}_1, \cdots, \hat{V}_p)$, which is normalized so that $\hat{V}^* S_{11} \hat{V} = I$.

(2) The cointegration rank r is the number of statistically non-zero eigenvalues. Given r, the likelihood ratio test statistic for the hypothesis $H(r)$ against $H(p)$ is given by the 'trace' statistic $\tau_{trace}(r)$ (abbreviated as $\tau(r)$):

$$\tau(r) = -T \sum_{i=r+1}^{p} \ln(1 - \hat{\lambda}_i). \tag{3.5}$$

The asymptotic distribution of $\tau(r)$ is nonstandard and quantile tables are given by Osterwald-Lenum (1992) based upon Monte Carlo simulations. In the case of there is no or little prior information about r, we might estimate r as follows: Denote by $\tau(i|1-\alpha)$ the $(1-\alpha)$ quantile of $\tau(i)$ and by $\hat{\tau}(i)$ be the observation of $\tau(i)$. If

$\hat{\tau}(0) < \tau(0|1-\alpha)$, we choose $\hat{r} = 0$. For $r = 1, \cdots, p-1$, let $\hat{r}$ be the first r such that

$$\hat{\tau}(r-1) > \tau(r-1|1-\alpha), \text{ and } \hat{\tau}(r) < \tau(r|1-\alpha),$$

and if there is no such r, then set $\hat{r} = p$. The estimates of the other parameters are obtained by the OLS by setting $\Pi = \alpha\hat{\beta}^*$ in equation (3.1).

The estimated eigenvalues and the corresponding eigenvectors, as well as observed trace statistics, are listed in Table 3.1. The variables of the models are indicated there. We estimate r in this paper based not only on the $\tau(r)$ statistic but also on the consideration of other aspects of data and the corresponding model.

Table 3.1 The Eigenvalues and the Eigenvectors and the Trace Statistics

	Eigenvalues					Eigenvalues			
	(0.112	0.062)				(0.053	0.051)		
	Eigenvectors		$p-r$	$\hat{\tau}$		Eigenvectors		$p-r$	$\hat{\tau}$
BCS	0.812	0.553	1	8.77	NDCS	1.000	-0.984	1	7.16
NDCS	-0.584	0.833	2	24.94	EIRW	-0.003	0.179	2	14.54
	Eigenvalues					Eigenvalues			
	(0.095	0.009)				(0.148	0.046)		
	Eigenvectors		$p-r$	$\hat{\tau}$		Eigenvectors		$p-r$	$\hat{\tau}$
Cls	-0.878	0.994	1	0.31	EIRW	0.945	0.997	1	6.42
AOR	0.479	-0.110	2	13.85	IIP	-0.327	0.079	2	28.24
	Eigenvalues								
	(0.090	0.053)				*The τ statistics*			
	Eigenvectors		$p-r$	$\hat{\tau}$		$p-r$	90%	95%	
BCS	1.000	-0.357	1	7.48		1	2.69	3.00	
EIRW	-0.019	0.934	2	12.78		2	13.33	15.00	

The quantiles of τ are from Table 1 in Osterwald-Lenum (1992)

A criterion for the lag length selection is that the resulting residuals are uncorrelated to a reasonable degree. The residual uncorrelation is checked by Portmanteau test in the following modified form given by Hosking (1980).

$$HK(s) = T^2 \sum_{j=1}^{s} \frac{1}{T-j} tr\{\hat{C}_{0j}\hat{C}_{00}^{-1}\hat{C}_{0j}^{*}\hat{C}_{00}^{-1}\}, \tag{3.6}$$

where

$$\hat{C}_{0j} = T^{-1} \sum_{t=j+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-j}^*.$$

Under the null hypothesis of uncorrelation, the distribution of this test statistic is approximated for large T and for $s >> a$ by χ^2 distribution with degrees of freedom $f = p^2(s-a)$ where a is the lag length of the model. For our cases of bivariate models, we choose $s = 18$ and the observed statistics are listed in Table 3.2. The results in Table 3.2 support that all the residuals in the models are reasonably uncorrelated.

Table 3.2 The HK-statistics and the p-values

	Hg-stat.	p-value		Hg-stat.	p-value
NDCS&BCS	49.5542	0.4110	AOR&CIs	62.9493	0.0725
EIRW&NDCS	70.9100	0.0170	IIP&EIRW	68.2062	0.0291
EIRW&BCS	61.4723	0.0916			

The HK-statistic is defined by (3.6).

The Gaussian assumption of the disturbance term is checked by applying the omnibus test for multivariate normality given by Doornik and Hansen (1994) to the residuals of the estimated models. Let R_r^* be the $p \times T$ matrix of the residuals with sample covariance matrix $F = (f_{ij})$. Create a matrix D with the reciprocals of the standard deviations on the diagonal,

$$D = diag(f_{11}^{-1/2}, \cdots, f_{pp}^{-1/2}), \tag{3.7}$$

and form the correlation matrix $C = DFD$. Define the transformed matrix of R_r by

$$R_c = HL^{-1/2}H^*DR_r^*, \tag{3.8}$$

where L is the diagonal matrix with the eigenvalues of C on the diagonal. The columns of H are the corresponding eigenvectors, such that $H^*H = I_p$ and $L = H^*CH$. Then we compute univariate skewness $\sqrt{b_{1i}}$ and kurtosis b_{2i} of each vector of the transformed $R_c^*, i = 1, \cdots, p$, where we follow the notations by Doornik and Hansen (1994). Under the null hypothesis of multivariate normal distribution of the residuals, the test statistic is asymptotically distributed as:

$$E_p = Z_1^* Z_1 + Z_2^* Z_2 \sim \chi^2(2p), \tag{3.9}$$

where $Z_1^* = (z_{11}, \cdots, z_{1p})$ and $Z_2^* = (z_{21}, \cdots, z_{2p})$. For $i = 1, \cdots, p$, the transformation for the skewness $\sqrt{b_{1i}}$ into z_{1i} is due to D'Agostino (1970), the kurtosis b_{2i} is transformed from a gamma distribution to χ^2, and then transformed into standard normal z_{2i} using the Wilson-Hilferty cubed root transformation. The observed test statistics E_p for all the models used in the analysis of Japanese data are listed in Table 3.3. Those test statistics seem to indicate that there is no significant departure from Gaussianity. The results in Tables 3.3 ensures us that we may proceed to the discussions on the one-way effect measurement on the basis of the proposed ECMs.

Table 3.3 The Ep-statistics and the p-values

	Hg-stat.	p-value		Hg-stat.	p-value
NDCS&BCS	11.4631	0.0218	AOR&CIs	2.8090	0.5903
EIRW&NDCS	7.7535	0.1010	IIP&EIRW	9.1498	0.0575
EIRW&BCS	10.0020	0.0404			

The Ep-statistic is defined by (3.9).

Remark 3.1. *Since twelve years of monthly data cannot be regarded as a large sample, to be conservative, we use $T - n_\psi$ instead of the sample size T in (2.9), (2.10) and (2.12).*

4 Empirical Analyses

We apply the Wald test to the study of the causal relationships among the business cycle indices of Kagawa Prefecture macroeconomic time series. The following analyses are conducted based on model (3.1). We investigated bivariate cases, the models are the interested pairs in the possible combinations of the six selected economic time series. The empirical examples would show some characteristics of the recent Kagawa Prefecture macroeconomy in view of the one-way causality.

For model (2.1), let $\hat{C}(e^{-i\lambda})$ be the adjoint of the matrix

$$I_p - (I_p + \hat{\alpha}\hat{\beta}^*)e^{-i\lambda} - \sum_{j=1}^{4} \hat{\Gamma}(j)(e^{-ij\lambda} - e^{-i(j+1)\lambda})$$

as given in Section 2. Then the measures of one-way effect from Y to X are estimated on the basis of the frequency response estimate $\hat{\Lambda}(e^{-i\lambda}) = \hat{C}(e^{-i\lambda})\hat{\Sigma}^{1/2}$ and the spectral density estimate $\hat{f}(\lambda) = \frac{1}{2\pi}\hat{\Lambda}(e^{-i\lambda})\hat{\Lambda}(e^{-i\lambda})^*$. As for the numerical evaluation of $D_\psi G$ in (2.5), we choose $h = 0.0001$; after having conducted evaluation of the Jacobian matrix for numerous choice including smaller h, we found that the results were sufficiently stable for $h = 0.0001$.

Figure 2 lists 5 plots of the estimated *FMO*. The estimates of cointegrating rank (r) and the *OMO* (M) as well as the Wald statistics W defined by (2.12) are also presented in the figures. The 95 per cent confidence intervals of the *OMO*, in case the null hypothesis of non-causality is rejected, are also listed in the corresponding figures. The *OMO* estimates are obtained by numerical integration of the estimated *FMO*'s by dividing $[0, \pi]$ into 200 equal intervals. For each of the models we calculate *FMO* for frequency points $\lambda_i = i\pi/200,\ i = 1, 2, \cdots, 200$. As for the number of division of $[0, \pi]$, we checked many cases of interval division up to 1200, and we found that the 200 equal-division of the interval $(0, \pi]$ is fine enough.

In view of Figure 2, notable findings are as follows:

- Plot (a1) shows that even the one-way effect from NDCS to the BCS is not significant at the 95 percent confidence level, the *OMO*=2.22 (p-value=0.14) is significant at about an 86 percent confidence level. The causal structure is similar to that of the effect from NDCS to CIs. The reverse, the effect of the BCS to NDCS, is not significant.
- The one-way effect from EIRW to NDCS (*OMO*=2.62) is significant, the 95 percent confidence region is (1.56, 3.68) [see plot (a2)]. Two peaks exist: the weak

one appears at about 0.8π frequency (nearly three months cycle), and the strong one appears at about 0.25π. The one-way effect does not seem long-run. The one-way effect from NDCS to EIRW is not observed.

- The one-way effect from EIRW to the BCS is OMO=2.52. Our Wald statistic (W=9.26) shows that it is significant [see plot (a3)]. The peaksis peak is reached in the frequency band $[0.2\pi, 0.4\pi]$, meaning the main effect comes at about the half year point.

- Plot (a4) shows that the one-way effect from IIP to EIRW is OMO=1.87 (with a p-value 0.06), which is significant at a 10 percent significance level but not long-run. The reverse of the effect is not observed. A weak effect from AOR to Cls is observed and is only long-run [see plot (a5)]. The one-way effect from Cls to AOR is not observed.

To summarize, the empirical analyses show that the Employment Index of Regular Workers is compararively active. The effect of Index of Industrial Production to Employment Index of Regular Workers is significant. The one-way effect of Active Opening Ratio to Clearings is observed but comparatively weak.

5 Concluding Remarks

In this paper we summarized the Wald test of one-way effect for cointegrated relations, and showed that causality hypotheses can be tested by standard asymptotically χ^2 distributed Wald statistics. The proposed method includes testing Granger's non-causality as an example of its multiple applications. Furthermore, the confidence set construction of a scalar OMO was shown by the use of the Wald test statistic.

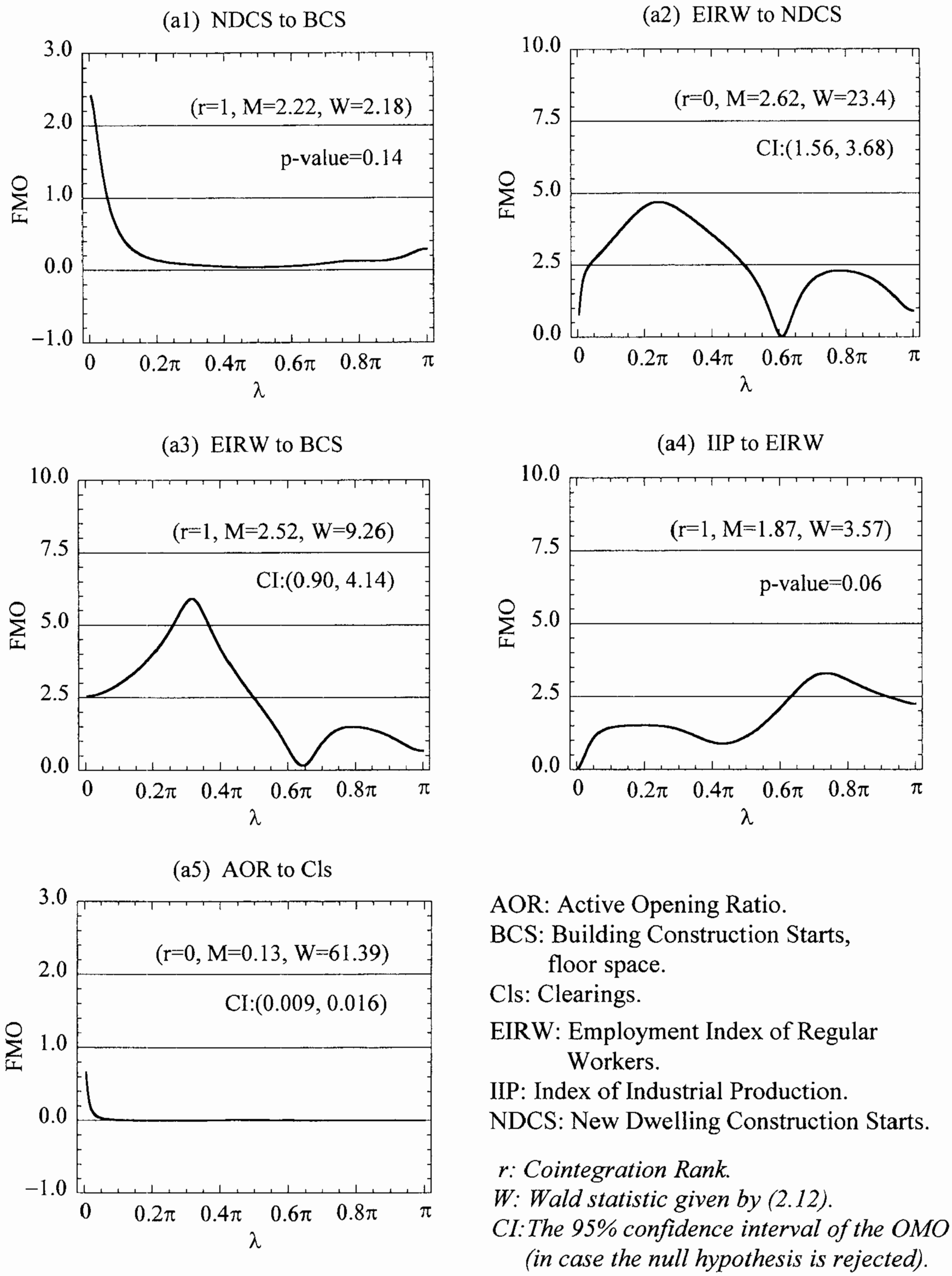


Figure 2. Estimated Masures of One-way Effect in Frequency Domain, Cointegration Rank, Testing Statistics, Overall Measure and Confidence Interval

We presented how the theory of the one-way effect is put into practice and how to interpret empirical evidence. The empirical analyses were conducted for six monthly macroeconomic series for the period of January 1987 through October 1998 in Kagawa Prefecture. The Wald test shows that the Employment Index of Regular Workers is comparatively active in the meaning of one-way effect. The Employment Index of Regular Workers seems affect almost all the other indices one-sided. At the time period we observed, only the effects from Index of Industrial Production to Employment Index of Regular Workers is significant and not long-run. The one-way effects of New Dwelling Construction Starts to Building Construction Starts (floor space) are mainly long-run at about an 86 percent confidence level. The one-way effects of Active Opening Ratio to Clearings are only long-run and weak.

In this paper, the analysis relies entirely upon "simple" causal relations, ignoring interaction with a third series in order for them to be formal "partial" causal measures. We know that partial causal measures explicitly take into account the presence of a third series effect, and its elimination might be more desirable for certain purposes if we start from a full model of a macroeconomy. Statistical inferences and empirical studies based on this approach will be dealt with in our forthcoming papers. For the purpose of comparison, we used the same VAR model in our investigation of the Kagawa Prefecture economic data. It shows that some models are not always the best suited for the data, so the one-way effect conclusion in such cases should be used carefully.

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