ISSN 2758-0520

THE INSTITUTE OF ECONOMICS RESEARCH

Working Paper Series

No. 260

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June 2023

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Abstract

We examine the impact of education subsidies on economic growth rates, considering the uncertainty surrounding individuals' lifetimes. We assume three educational inputs - parental private investment, education subsidies as a complement to parental investment, and public schooling - affect the accumulation of children's human capital. Our findings demonstrate that implementing education subsidies alongside public schooling can enhance growth rates within existing budget constraints. This effect is especially pronounced in aging economies with a significant proportion of the population living into old age.

Keywords: Education subsidies; Public education; Uncertain lifetime; OLG model

JEL: H31; H52; J10

[*]This manuscript is a revised version of the prior working paper titled "The effect of education subsidies in an aging economy" (originally circulated as Discussion Papers In Economics and Business 05-30, Osaka University), which I wrote during my time as a graduate student at Osaka University. I am grateful to Koichi Futagami, Ryo Horii, Kazuo Mino, Akira Momota, Tetsuo Ono, and Akira Yakita for their valuable comments. All remaining errors are, of course, my own.

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1 Introduction

Industrialized nations are experiencing a rapid aging phenomenon. When aiming to sustain pension systems that are financed through pay-as-you-go schemes while offering moderate benefits, the tax burdens on the younger working generation become significantly heavier. Consequently, young working households, who allocate a portion of their after-tax income towards an extended period of retirement, find it challenging to allocate substantial resources towards their children's upbringing and education. This research investigates the impact of education subsidies, which serve as a complement to parental investments in education, on economic growth, taking into account the effects of population aging.

Previous studies, including those conducted by Zhang and Casagrande (1998), Kaganovich and Zilcha (1999), Wigger (2004), Chen (2015), Morimoto and Tabata (2020), and Chen and Miyazaki (2022), among others, have explored the effects of education subsidies on the economic growth. Kaganovich and Zilcha (1999) reveal that the introduction of education subsidies can lead to higher growth rates only under specific conditions: (i) when parents possess low levels of preference for private investments in their children's education, and (ii) when parents exhibit moderate preference levels for children, while the government maintains substantial revenue for education (relative to pension revenue). In other cases, the negative effects of subsidies, which crowds out private investments by parents, are so significant that the introduction of education subsidies either yields no growth effect or even negatively impacts the growth rate.

Our motivation lies in examining the effectiveness of education subsidies in countries facing the challenge of limited private investments in their children's education due to population aging. By incorporating an uncertain lifetime into a model based on Kaganovich and Zilcha (1999), we demonstrate that unless the scale of the education budget is excessively constrained, the government can enhance the economic growth rate of an aging economy within the same education budget by implementing education subsidies that complement household educational investments and public education.

The remainder of this paper is structured as follows: Section 2 introduces the model, Section 3 compares the growth effects of two education policies, and Section 4 concludes with final remarks.

2 The model

We consider an overlapping-generations model of endogenous growth, incorporating an uncertain lifetime. A representative individual's life is divisible into three periods: a childhood and a young-working period (each with fixed duration), and a retirement period (of uncertain length). Individuals are alive at the beginning of the third period with probability $p \in (0, 1]$. Let N_t denote the number of working-aged individuals, who have n children, in period t. Savings of individuals who have died at the onset of the third period are distributed among the retired individuals as an actuarially fair annuity.[1] The expected rate of return to savings is $(1 + \rho_{t+1}) = \frac{(1+r_{t+1})}{p}$, where $(1+r_{t+1})$ is the return of direct holdings of capital. The government supplies education to children and social security benefits to retired individuals by taxing the labor income of young-working individuals. First, we examine an Education-Subsidy Policy (ESP), by which the government provides both ed-

[1]A simplified version of Blanchard's (1985) model.

ucation subsidies and public schools. Secondly, we show the equilibrium of a Public-School Policy (PSP) in which the government supplies public schools alone. Thereafter, in the next section, we compare the two growth rates.

In childhood, individuals accumulate only human capital. Young individuals receive a wage income that is taxed away. They divide their income among education expenditures for their children, their current consumption, and investing in annuities for their post-retirement consumption. Subsequently, living individuals obtain principal and interest from their annuities and consume them with their pension benefits after retirement.

Let h_{t+1} be the human capital level of each individual born at time t; they are called generation $t+1$. Human capital is accumulated according to:

$$h_{t+1} = (e_t + \nu_t)^{\gamma}(e_t^g)^{1-\gamma}, \tag{1}$$

where e_t is the private education input by their parents, such as notebooks and tutors, ν_t is the education subsidy given by the government, and e_t^g is the public-school quality provided by the government.[2] e_t, ν_t, and e_t^g are all per-capita values. In that equation, $\gamma \in (0, 1)$ denotes the efficiency of education input provided privately. Subsidies are used directly for human capital accumulation as in the methods of Kaganovich and Zilcha (1999). Individuals determine their children's amount of private investment per capita, e_t, taking the value of ν_t and e_t^g as given.

[2]We call e_t^g "public-school quality", as do Glomm and Ravikumar (1992), to distinguish strictly between ν_t, which is subsidized to private parental investment, and e_t^g. We can regard e_t^g not only public schools but also other public educational investment like libraries, museums, and so on.

The lifetime utility function of generation t is represented as[3]

$$u_t = \ln c_t^y + p\ln c_{t+1}^o + \delta n \ln h_{t+1}, \tag{2}$$

where c_t^y and c_{t+1}^o are the amount of consumption in young and old, and δ represents the degree of preferences over the total amount of childrens' human capital.

Budget constraints of members of generation t when young and retired are given, respectively, as

$$(1-\tau_t-\omega)w_t h_t = c_t^y + ne_t + a_t \tag{3}$$

and

$$(1+\rho_{t+1})a_t + T_{t+1} = c_{t+1}^o, \tag{4}$$

where τ_t is the social-security-tax rate, ω is the education-tax rate, w_t is the wage rate, a_t is the annuity, and T_{t+1} is the pension benefit.

By solving the optimization problems of individuals outlined in equations (1)-(4), we derive the optimal values as follows

$$e_t = \frac{\gamma\delta}{(1+p+\gamma\delta n)} I_t - \frac{(1+p)}{(1+p+\gamma\delta n)} \nu_t, \tag{5}$$

$$c_t^y = \frac{1}{(1+p+\gamma\delta n)} I_t,$$

[3]With an uncertain lifetime, $p \in (0,1]$, this utility form is employed by Yakita (2001) and Pecchenino and Pollard (2002), among others.

$$c^o_{t+1} = \frac{p}{(1+p+\gamma\delta n)}(1+\rho_{t+1})I_t,$$

and

$$a_t = \frac{p}{(1+p+\gamma\delta n)}I_t - \frac{T_{t+1}}{(1+\rho_{t+1})}, \tag{6}$$

where

$$I_t \equiv (1-\tau_t-\omega)w_t h_t + n\nu_t + \frac{T_{t+1}}{(1+\rho_{t+1})}. \tag{7}$$

Aggregate production at time t is given as $Y_t = AK_t^\alpha(h_t N_t)^{1-\alpha}$, where Y_t, A, K_t, and $\alpha\in(0,1)$ respectively denote the aggregate output, the productivity parameter, the physical capital that fully depreciates in the production process, and the share of physical capital. Because the factor markets are presumed to be perfectly competitive, the firms take factor prices as given: $w_t = A(1-\alpha)(\frac{K_t}{h_t N_t})^\alpha$, and $(1+r_t) = A\alpha(\frac{K_t}{h_t N_t})^{\alpha-1}$.

The government allocates the $\mu \in [0,1)$ portion of the education-tax revenue to education subsidies and the $(1-\mu)$ portion of it to public schools.[4] Here, μ is treated as a predetermined parameter and is constant over time. Budget constraints of the government per child at time t are

Education Subsidy; $$\nu_t = \frac{\mu\omega}{n}w_t h_t, \tag{8}$$

Public School; $$e^g_t = \frac{(1-\mu)\omega}{n}w_t h_t. \tag{9}$$

[4] When $\mu = 0$, it means PSP is operated.

Social security payments are specified as a replacement rate, $\phi \in (0,1)$, on current workers' wage income, as

$$T_t = \phi w_t h_t. \tag{10}$$

Total pension benefits must be balanced by pension revenue:

$$T_t p N_{t-1} = \tau_t w_t h_t N_t. \tag{11}$$

From (10) and (11), the social-security-tax rate is determined as

$$\tau_t = \frac{p\phi}{n} \equiv \tau, \tag{12}$$

which is constant over time. Note that this contribution rate is increasing in the degree of aging, p.

Incorporating equations (1), (5)-(10), (12), the expected rate of return to savings such as $(1+\rho_{t+1}) = \frac{(1+r_{t+1})}{p}$, and the capital market-clearing condition, $K_{t+1} = a_t N_t$, the representation of the human capital level for generation $t+1$ emerges as follows:

$$h_{t+1} = (\frac{s^*_\nu\{1+\frac{(1-\alpha)}{\alpha}\tau\}\gamma\delta}{p})^\gamma(\frac{(1-\mu)\omega}{n})^{1-\gamma} w_t h_t \equiv h^*_\nu w_t h_t,$$

where

$$s^*_\nu \equiv \frac{p\{1-\tau-(1-\mu)\omega\}}{\{(1+p+\gamma\delta n)+(1+\gamma\delta n)\frac{(1-\alpha)}{\alpha}\tau\}},$$

and

$$h^*_\nu \equiv (\frac{s^*_\nu\{1+\frac{(1-\alpha)}{\alpha}\tau\}\gamma\delta}{p})^\gamma(\frac{(1-\mu)\omega}{n})^{1-\gamma}.$$

Consequently, the per-capita growth rate at time t is constant over time:

$$(1+g_{\nu,t}) \equiv \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}} = \frac{A(1-\alpha)}{n} s_\nu^{*\alpha}(h_\nu^* n)^{1-\alpha} \equiv (1+g_\nu). \qquad : ESP$$

When no education subsidies are applicable, $\mu = 0$, the per-capita growth rate at time t is given as

$$(1+g_t) \equiv \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}} = \frac{A(1-\alpha)}{n} s^{*\alpha}(h^* n)^{1-\alpha} \equiv (1+g), \qquad : PSP$$

where

$$s^* \equiv \frac{p(1-\tau-\omega)}{\{(1+p+\gamma\delta n)+(1+\gamma\delta n)\frac{(1-\alpha)}{\alpha}\tau\}},$$

and

$$h^* \equiv \left(\frac{s^*\{1+\frac{(1-\alpha)}{\alpha}\tau\}\gamma\delta}{p}\right)^\gamma \left(\frac{\omega}{n}\right)^{1-\gamma}.$$

3 Education subsidies vs. Public schools

We shall set the ratio of the growth rate of ESP, $(1+g_\nu)$, to the growth rate of PSP, $(1+g)$, as

$$G(p) \equiv \frac{(1+g_\nu)}{(1+g)} = (1-\mu)^{(1-\gamma)(1-\alpha)}\left(\frac{1-\tau-(1-\mu)\omega}{(1-\tau-\omega)}\right)^{\alpha+(1-\alpha)\gamma}. \qquad (13)$$

Inferring that the social-security-tax rate, τ, is influenced by the degree of aging, p, we obtain the following proposition.

Proposition.

(1) When the education-tax rate is higher than ω_H, the ESP implies a higher growth rate than the PSP.

(2) When the education-tax rate is medium, as $\omega_L \leq \omega \leq \omega_H$, the ESP implies:

(2a) a higher growth rate if the degree of aging is higher than $\hat{p}$

(2b) a lower growth rate if the degree of aging is lower than $\hat{p}$

than the PSP.

(3) When the education-tax rate is lower than ω_L, the ESP implies a lower growth rate than the PSP.

These threshold values are:

$$\omega_H = \frac{1 - (1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}}{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}}} \in (0,1),$$

$$\omega_L = \frac{(1-\frac{\phi}{n})\{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\}}{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}}} \in (0,1),$$

$$\hat{p} = \frac{n}{\phi}\frac{[(1-\omega)-\{1-(1-\mu)\omega\}(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}]}{\{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\}} \in (0,1).$$

Proof. See Appendix. □

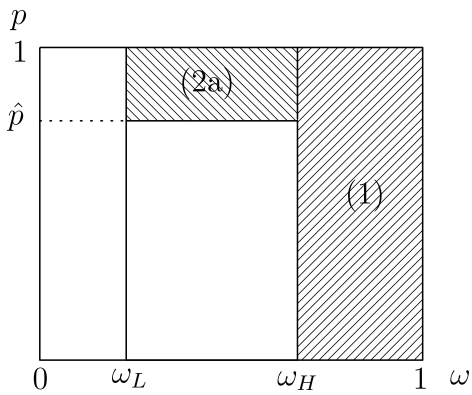

Figure 1: The ESP growth rate is higher than that of PSP in the shaded area.

If government expenditures on public-school investment significantly surpass private investment, reallocating a portion of education revenue from public-school investment to education subsidies that complement private investment can result in a more efficacious allocation and engender elevated rates of growth. Our theoretical framework demonstrates that in economies where parents' after-tax income is diminished due to higher education-tax rates (i.e., Proposition (1)) or where parents aim to allocate a larger portion of their income towards longer retirement periods despite facing heavier social-security burdens (i.e., Proposition (2a)), parental investment becomes comparatively smaller. In other words, unless the education-tax rate is sufficiently low ($0 < \omega < \omega_L$), parental private investment remains considerably smaller than public-school investment in aging economies. Under such circumstances, the introduction of education subsidies enhances the efficiency of human capital accumulation and generates a higher growth rate.[5]

The implications of our findings closely align with those of Kaganovich and Zilcha (1999), as they demonstrate that when parents do not adequately

[5]We can readily ascertain the existence of an optimal allocation of μ by partial differentiation of Equation (13).

engage in private investment, the use of education subsidies as complements to private investment results in higher growth rates while maintaining identical levels of tax revenue.[6] However, despite the noteworthy significance of threshold levels of parental preferences for their children's education, as evidenced in the study conducted by Kaganovich and Zilcha (1999), the levels of such preferences exhibit considerable variations among individuals and pose challenges in terms of measurement. Our contribution lies in the demonstration that two observable parameters, namely the degree of economic aging and the education-tax rates, serve as decisive thresholds for determining the effects of subsidies. This is achieved by incorporating the element of uncertain lifetime into the existing model of Kaganovich and Zilcha (1999).

4 Concluding remarks

We have conducted an analysis to investigate the impact of introducing education subsidies on growth rates, while considering the element of uncertain lifetime. Our findings illustrate that the implementation of subsidies leads to higher growth rates while simultaneously ensuring that tax revenue remains unchanged in economies experiencing the phenomenon of population aging.

[6] When parental investment is sufficiently large because of a lower education-tax rate and a lower aging degree (the lower left area in Fig. 1), the ESP engenders a lower growth rate than the PSP. This result differs from that of Kaganovich and Zilcha (1999), in which the introduction of education subsidies has no effect on the growth rate if parents invest much in their children because of larger preference for their children.

Appendix.

Proof. Using $\tau = \frac{p\phi}{n}$, we recognize that the ratio, $G(p)$, is increasing in aging degree, p, as

$$sign(\frac{\partial G(p)}{\partial p}) = sign(\frac{\phi}{n}\frac{\{\alpha + (1-\alpha)\gamma\}\mu\omega}{\{1-\tau-(1-\mu)\omega\}(1-\tau-\omega)}) > 0.$$

Initially, $p = 0$. The education-tax rate, which is satisfied with

$$G(p = 0) \geq 1,$$

is expressed as

$$\omega \geq \frac{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}}{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}}} \equiv \omega_H.$$

Therefore, when the education-tax rate is higher than ω_H, $G(p) > 1$ is always satisfied in $p \in (0, 1]$. The ESP always engenders a higher growth rate.

Secondly, we check at $p = 1$. The range of the education-tax rate, which is satisfied with

$$G(p = 1) \geq 1,$$

is given as

$$\omega \geq \frac{(1-\frac{\phi}{n})\{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\}}{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}}} \equiv \omega_L.$$

That is, when the education-tax rate is $\omega_L \leq \omega \leq \omega_H$, $G(p) > 1$ is satisfied at least at $p = 1$. In contrast, when the education-tax rate is $\omega < \omega_L$, $G(p)$ remains less than 1 over $p \in (0, 1]$. Consequently, when the education-tax rate is lower than ω_L, the PSP always leads to a higher growth rate.

Finally, in the case where the education-tax rate is medium, as $\omega_L \leq \omega \leq \omega_H$, the value of $G(p)$, which is an increasing function in p, is less than 1 at

$p = 0$ and more than 1 at $p = 1$. Here, a threshold value of p is satisfied with $G(p) = 1$. This threshold value is expressed as

$$\hat{p} = \frac{n}{\phi} \frac{\left[(1-\omega) - \{1-(1-\mu)\omega\}(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\right]}{\left\{1-(1-\mu)^{\frac{1}{\alpha+\gamma(1-\alpha)}-1}\right\}} \qquad (\omega_L \leq \omega \leq \omega_H).$$

Accordingly, when the economy's degree of aging is lower than $\hat{p}$, the value of $G(p)$ remains less than 1. When the degree of aging is higher than $\hat{p}$, $G(p)$ is greater than 1. Consequently, only in aging economies ($\hat{p} < p$) does ESP lead to a higher growth rate when the education-tax rate is medium, as $\omega_L \leq \omega \leq \omega_H$. □

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