

A Model of Hot-Sector Generation

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Abstract

Possible existence of “hot-sector generations” above the well known 3 generation bound is investigated on the basis of a model of leptons and quarks, which is based on the Harari and Shupe’s one. Our model predicts the existence of $3 + 1$ generations above the ordinary “cold-sector” 3 generations. Majorana neutrinos are introduced to realize the heavy neutrino masses in hot-sector generations. Properties of heavy neutrinos are also discussed.

1 Introduction

The 3 generation structure appearing in low energy region seems to bring suggestions concerning to the deeper level of nature[1]. Is the number of generations restricted to just 3? A certain kind of models predict possible existence of “hot-sector generations” above the 3 generation structure. In what form can the hot-sector generations exist? This paper is concerning to this problem.

The concept of hot-sector generation had been proposed by Maki[2] in a consideration of Blokhintsev type[3] for the meaning of presently appearing generation structure. He discussed that the “standard” particle picture will not hold in very high-energy region, and the introduction of very heavy particles would upset easily and drastically the standard physical features of the fields participating in the low-lying generations as a whole, leading the standard model to be almost meaningless. Then, the single question “how many generations are there in nature” should be divided into two similar questions as regards to “the cold- and hot-sector generations”[2], respectively.

Thus, the famous decision[4] of the generation number resulting from $Z \rightarrow \bar{\nu}_\ell \nu_\ell$ experiment should be interpreted that the number of “cold-sector generations” is just 3. Then, what reason is there behind this fact. What structure is expected for the hot-sector generations? It should be emphasized that in such energy region as hot-sector generations, the dynamics and particle picture should

be drastically altered from ordinary field theory, then it is meaningful to investigate a simple model to realize the characteristic features of phenomena. These circumstances should be compared with early stage of quark model [5, 6]. We will suppose that our model is concerning to the sub-structure, which suggests a guiding principle to build a model in the framework of GUTs structure[7].

2 Schematical model of generations

For definiteness, we will refine and summarize the essence of our previous work on generation structure[8]. Our model of generations is based on the rishon model[9] of Harari and Shupe, where all leptons and quarks are 3 body system of rishons T with charge 1/3 and V with charge 0, and freedom of color are realized by their configuration.

In our model, the rishons are defined as quantum states transforming like the fundamental (and its conjugate) representations of the group $SU_3(H) \times SU_3(C) \times SU_3(R)$. We first introduce the rishons and their quantum numbers $G = \pm 1/3$ for hypercolor, $Y = \pm 1/3$ for color and $Z = \pm 1/3$ for ‘‘R-color’’, respectively. The fundamental representation of $SU_3(H)$ is characterized by the $G = 1/3$ quantum number ($G=-1/3$ for the 3 conjugate representation) while the fundamental representation of $SU_3(C)$ is characterized by the $Y=1/3$ quantum number and that of $SU_3(R)$ is characterized by the $Z=1/3$. The number of electric charge is represented as

$$Q = \frac{1}{2}(Y + G) \quad (1)$$

We represent rishon as $R(\alpha i; \lambda)$ where α denotes the hypercolor and i the color, λ the R-color indices of the state. The internal quantum number of R_α state is $G=1/3$ and represent the rishon state while R^α is $G=-1/3$ and anti-rishon state. Similarly, the quantum number of R_i is $Y=1/3$ and R_λ is $Z=1/3$ etc. The R^α state with $Z=-1/3$ is represented by symbol \bar{R} , and called as ‘‘pre-generation state’’. The correspondence to the T and V states is given as

$$R_{\alpha i} = T_{\alpha i}, R^{\alpha i} = \bar{T}^{\alpha i}, R_\alpha^i = V_\alpha^i, R_i^\alpha = \bar{V}_i^\alpha \quad (2)$$

We now contract three rishon state on their hypercolor indices, and getting a singlet in hypercolor:

$$\Psi(ijk; \lambda\mu\nu) = \sum_{\alpha\beta\gamma} |R_\alpha(i\lambda)R_\beta(j\mu)R_\gamma(k\nu)\rangle \epsilon^{\alpha\beta\gamma}, \quad (3)$$

$$\bar{\Psi}(ijk; \lambda\mu\nu) = \sum_{\alpha\beta\gamma} \langle R^\alpha(i\lambda)R^\beta(j\mu)R^\gamma(k\nu) | \epsilon_{\alpha\beta\gamma}, \quad (4)$$

where suffix ijk and $\lambda\mu\nu$ represent upper or lower ones. The hypercolor quantum number G of such an object $\psi(ijk; \lambda\mu\nu)$ will be $G=1$, and that of $\bar{\psi}(ijk; \lambda\mu\nu)$ is $G=-1$. Contracting with respect to R-color indices, we obtain

$$\Psi_{\sigma}(ijk) = \sum_{\lambda\mu\nu\rho} \Psi(ijk)_{\lambda\mu}{}^{\nu} \epsilon^{\lambda\mu\rho} \epsilon_{\nu\rho\sigma} \quad (5)$$

$$\bar{\Psi}_{\sigma}(ijk) = \sum_{\lambda\mu\nu\rho} \bar{\Psi}(ijk)_{\lambda\mu}{}^{\nu} \epsilon^{\lambda\mu\rho} \epsilon_{\nu\rho\sigma} \quad (6)$$

where the suffix σ represent the generation label. The contraction of color indices leads to color triplets having the internal quantum number of the U type quarks together with generation label σ .

$$\begin{aligned} U_{\sigma,m} &= \sum_{ijkl} \Psi_{\sigma,ij}{}^k \epsilon^{ijl} \epsilon_{klm} \\ &= \sum_{ijkl} \sum_{\lambda\mu\nu\rho} \sum_{\alpha\beta\gamma} |R_{\alpha i\lambda} R_{\beta j\mu} R_{\gamma}^{k\nu}\rangle \epsilon^{ijl} \epsilon_{klm} \epsilon^{\alpha\beta\gamma} \epsilon^{\lambda\mu\rho} \epsilon_{\nu\rho\sigma} \\ &= \sum_{ijkl} \sum_{\lambda\mu\nu\rho} \sum_{\alpha\beta\gamma} |T_{\alpha i\lambda} T_{\beta j\mu} V_{\gamma}^{k\nu}\rangle \epsilon^{ijl} \epsilon_{klm} \epsilon^{\alpha\beta\gamma} \epsilon^{\lambda\mu\rho} \epsilon_{\nu\rho\sigma} \end{aligned} \quad (7)$$

where m represent the color label. The quantum number of this state is given as $G=1$, and $Y=1/3$, or equivalently $Q=2/3$, and $Z=1/3$. The configuration of D type quarks is also given by

$$\begin{aligned} D_{\sigma,m} &= \sum_{ijkl} \bar{\Psi}_{\sigma,ij}{}^k \epsilon^{ijl} \epsilon_{klm} \\ &= \sum_{ijkl} \sum_{\lambda\mu\nu\rho} \sum_{\alpha\beta\gamma} \langle R_{\sigma\lambda}^{\alpha} R_{j\mu}^{\beta} R^{\gamma k\nu} | \epsilon^{ijl} \epsilon_{klm} \epsilon_{\alpha\beta\gamma} \epsilon^{\lambda\mu\rho} \epsilon_{\nu\rho\sigma} \\ &= \sum_{ijkl} \sum_{\lambda\mu\nu\rho} \sum_{\alpha\beta\gamma} \langle \bar{V}_{\sigma\lambda}^{\alpha} \bar{V}_{j\mu}^{\beta} \bar{T}^{\gamma k\nu} | \epsilon^{ijl} \epsilon_{klm} \epsilon_{\alpha\beta\gamma} \epsilon^{\lambda\mu\rho} \epsilon_{\nu\rho\sigma} \end{aligned} \quad (8)$$

where $G=-1$, $Y=1/3$, $Q=-1/3$, and $Z=1/3$.

The singlet in hypercolor and color corresponds to lepton state,

$$\begin{aligned} \ell_{\sigma} &= \sum_{ijk} \bar{\Psi}_{\sigma}{}^{ijk} \epsilon_{ijk} \\ &= \sum_{ijk} \sum_{\lambda\mu\nu\rho} \sum_{\alpha\beta\gamma} \langle \bar{T}_{\lambda}^{\alpha,i} \bar{T}_{\mu}^{\beta,j} \bar{T}^{\gamma k\nu} | \epsilon_{ijk} \epsilon_{\alpha\beta\gamma} \epsilon^{\lambda\mu\rho} \epsilon_{\nu\rho\sigma} \end{aligned} \quad (9)$$

with $G=-1$, $Y=-1$, $Q=-1$, and $Z=1/3$.

Similarly, the configuration on neutrino with generation label σ is given by

$$\nu_{\sigma} = \sum_{ijk} \Psi_{\sigma}{}^{ijk} \epsilon_{ijk}$$

$$= \sum_{ijk} \sum_{\lambda\mu\nu\rho} \sum_{\alpha\beta\gamma} |V_{\alpha\lambda}^i V_{\beta\mu}^j V_{\gamma}^{k\nu}\rangle \epsilon_{ijk} \epsilon^{\alpha\beta\gamma} \epsilon^{\lambda\mu\rho} \epsilon_{\nu\rho\sigma} \quad (10)$$

where $G=1$, $Y=-1$, $Q=0$, and $Z=1/3$.

Thus, in the framework of geometrical model[10], the generation label can be introduced without any ambiguity[11].

3 Structure of “hot-sector generations”

In our model, the 3 generation structure of “cold-sector” generations is represented by $\Psi_{\lambda,\mu}^\nu$ representation of $SU_3(R)$ group. It should be noted that there appears further configurations $\Psi_{\lambda,\mu,\nu}$, $\Psi_{\lambda}^{\mu,\nu}$ and $\Psi^{\lambda,\mu,\nu}$. What is meant by the existence of these configurations? The most natural interpretation is to identify them to the hot-sector generations. That is, there are 3 generations in hot-sector, which is represented by $\Psi_{\lambda}^{\mu,\nu}$. Further, there is the other configuration, which is represented as $\Psi^{\lambda,\mu,\nu}$. This will mean the existence of further one hot generation. That is, our model suggests the following generation structure:

$\Psi_{\lambda,\mu,\nu}$	frozen sector	$Z = 1$
$\Psi_{\lambda,\mu}^\nu$	3 cold sector generations	$Z = 1/3$
$\Psi_{\lambda}^{\mu,\nu}$	3 hot sector generations	$Z = -1/3$
$\Psi^{\lambda,\mu,\nu}$	1 hot sector generation	$Z = -1$

The $\Psi_{\lambda,\mu,\nu}$ configuration which contains no pre-generation state \hat{R} should be interpreted that its sector has been frozen by some reason. The $\Psi_{\lambda}^{\mu,\nu}$ and $\Psi^{\lambda,\mu,\nu}$ configurations represent the hot-sector generations. That is, $\mathbf{3} + \mathbf{1}$ structure of hot-sector generations is expected in our model. Then, what is meant by the “hot-sector” and “frozen sector” generations? We will stand on the view-point that the rishon system is the one beyond the ordinary quantum field theory, and we have treated only classification symmetry without treating the details of dynamics. The new dynamics may be related to the quantum field theory with a specific structure and principle, or further may be beyond the quantum theory though it seems to be extremely applicable. In the present stage, however, it is difficult to find out yet to be known new dynamics in the complete form. It is important to note that the new dynamics should be the one to lead to the standard model effectively in an appropriate energy region. From this view-point, the fruits of field theoretical approach to sub-system should be remarked[12]. Especially, it is known that the possession of a certain kind of symmetry, ie. chiral symmetry and/or supersymmetry, in the gauge theory of composite particle formation leads to the realization of the light fermion. Some models based on this mechanism are proposed, which predict the existence of heavy eccentric particles[13]. It

is probable that the existence of cold and hot-sector generations in our model is founded by making use of such mechanism. The constitution of theory containing hot-sector generation will be forced to take a form of mosaic of quantum field theory, because that generations are expected to be beyond the ordinary quantum field theory. In such a practice, the meaning of the frozen-sector may be also clarified. It is probable that the frozen sector is understood as ghost in a space with indefinite metric, or it does not form the bound state in the present vacuum, though it appears in the specific vacuum such as in early universe as bound states, etc. As a step to approach to these problems, we will examine a possible model of the hot-sector generations in the framework of present field theory.

4 Neutrino mass in hot-sector generations

Our model predict possible existence of hot-sector generations with $\mathbf{3} + \mathbf{1}$ structure. However, the result of experiment of $Z \rightarrow \bar{\nu}_\ell \nu_\ell$ shows that the number of neutrinos concerning to this process is just 3 [4]. That is, the number of neutrinos with mass below $M_Z/2$ is restricted to 3. Then, the neutrino masses of possible hot-sector generations should be above $M_Z/2$, so far as the same simple generation structure as cold-sector is maintained[14].

As is well known, the smallness of ordinary neutrino mass is nicely explained by the see-saw mechanism. If this mechanism is realized in the neutrinos of cold-sector generations, it is natural to suppose that a certain kind of see-saw mechanism is also realized in some neutrinos belonging to the hot-sector generations. What mechanism to satisfy the neutrino mass bound appears in that case?

As a basis for the construction of our scheme, let us consider the D-M (Dirac-Majorana) mass term [15]-[18] in the simplest case of one generation labeled by the generation subscript σ . We have

$$\begin{aligned} \mathcal{L}^{D-M} &= -\frac{1}{2} m_{\sigma L} \overline{(\nu_{\sigma L})^c} \nu_{\sigma L} - m_{\sigma D} \bar{\nu}_{\sigma R} \nu_{\sigma L} \\ &\quad -\frac{1}{2} m_{\sigma R} \bar{\nu}_{\sigma R} (\nu_{\sigma R})^c + h.c. \\ &= -\frac{1}{2} \begin{pmatrix} (\nu_{\sigma L})^c \\ \nu_{\sigma R} \end{pmatrix} M \begin{pmatrix} \nu_{\sigma L} \\ \nu_{\sigma R}^c \end{pmatrix} + h.c. \end{aligned} \quad (11)$$

Here

$$M = \begin{pmatrix} m_{\sigma L} & m_{\sigma D} \\ m_{\sigma D} & m_{\sigma R} \end{pmatrix}, \quad (12)$$

where $m_{\sigma L}, m_{\sigma D}, m_{\sigma R}$ are parameters. For a symmetrical matrix M we have

$$M = UmU^\dagger, \quad (13)$$

where $U^\dagger U = 1$ and $m_{jk} = m_j \delta_{jk}$. From Eqs. (11) and (13) we have

$$\mathcal{L}^{\text{D-M}} = -\frac{1}{2} \sum_{\alpha=1}^2 m_{\sigma\alpha} \bar{\chi}_{\sigma\alpha} \chi_{\sigma\alpha}, \quad (14)$$

where

$$\begin{aligned} \nu_{\sigma L} &= \cos \theta_\sigma \chi_{\sigma 1L} + \sin \theta_\sigma \chi_{\sigma 2L}, \\ (\nu_{\sigma R})^c &= -\sin \theta_\sigma \chi_{\sigma 1L} + \cos \theta_\sigma \chi_{\sigma 2L}. \end{aligned} \quad (15)$$

Here $\chi_{\sigma 1}$ and $\chi_{\sigma 2}$ are fields of Majorana neutrinos with masses $m_{\sigma s}$ (a “small” mass), $m_{\sigma B}$ (a “Big” mass), respectively. The masses $m_{\sigma s}$ and $m_{\sigma B}$ and the mixing angle θ_σ are connected to the parameters $m_{\sigma L}, m_{\sigma D}$ and $m_{\sigma R}$ by the relations

$$\begin{aligned} m_{\sigma s} &= \frac{1}{2} |m_{\sigma R} + m_{\sigma L} - a_\sigma|, \\ m_{\sigma B} &= \frac{1}{2} |m_{\sigma R} + m_{\sigma L} + a_\sigma|, \\ \sin 2\theta_\sigma &= \frac{2m_{\sigma D}}{a_\sigma}, \quad \cos 2\theta_\sigma = \frac{m_{\sigma R} - m_{\sigma L}}{a_\sigma}, \end{aligned} \quad (16)$$

where

$$a_\sigma = \sqrt{(m_{\sigma R} - m_{\sigma L})^2 + 4m_{\sigma D}^2}. \quad (17)$$

It should be noted that the relations Eq. (16) are exact.

4.1 Heavy neutrinos in hot-sector generation

Let us assume now that

$$m_{\sigma L} = m_{\sigma 0}, \quad m_{\sigma D} \simeq m_{\sigma F}, \quad m_{\sigma R} \gg m_{\sigma F}, \quad (18)$$

where $m_{\sigma F}$ is a typical mass of the leptons and quarks of the generation labeled by the subscript σ . Here, $m_{\sigma 0}$ should have an appropriate value above M_Z . If we assume, as a proto type of typical case, that $m_{\sigma B} \simeq 10^{19}$ GeV (Planck mass) and $m_{\sigma 0} = 100\text{GeV}$, then we see that

$$m_{\sigma s} \simeq 100\text{GeV}, \quad m_{\sigma B} \simeq 10^{19}\text{GeV}. \quad (19)$$

Are the heavy neutrinos stable? They may decay into particles in cold sector through a very small mixing of hot-sector generations with cold sector ones, or through the interaction of heavy gauge bosons appearing in GUTs.

4.2 See-saw mechanism in cold-sector generations

Instead of Eq. (18), if we assume[15]-[18]

$$m_{\sigma L} = 0, \quad m_{\sigma D} \simeq m_{\sigma F}, \quad m_{\sigma R} \gg m_{\sigma F}, \quad (20)$$

it leads to well known see-saw mechanism

$$m_{\sigma s} \simeq \frac{m_{\sigma F}^2}{m_{\sigma R}}, \quad m_{\sigma B} \simeq m_{\sigma R}, \quad \theta_i \simeq \frac{m_{\sigma D}}{m_{\sigma R}} \quad (21)$$

Thus, if the conditions Eq. (20) are satisfied, the particles with definite masses are split to a very light Majorana neutrino with mass $m_{\sigma s} \ll m_{\sigma F}$ and a very heavy Majorana particle with mass $m_{\sigma B} \simeq m_{\sigma R}$. The current neutrino field $\nu_{\sigma L}$ practically coincides with $\chi_{\sigma 1L}$ and $\chi_{\sigma 2} \simeq \nu_{\sigma R} + (\nu_{\sigma R})^c$, because θ_i is extremely small. That is, we have assumed such scheme that in D-M mass term Dirac masses are of order of usual fermion masses, the right-handed Majorana masses, responsible for lepton numbers violation, are extremely large and the left-handed Majorana masses are equal zero. In such a scheme neutrinos are Majorana particles with masses much smaller than masses of the other fermions.

5 Discussion

In this paper, we have proposed a model of realization of hot-sector generation. In our model, the neutrino mass of hot-sector generations is realized by a certain kind of see-saw mechanism, in which Majorana mass term of $m_{\sigma L} \overline{(\nu_{\sigma L})^c} \nu_{\sigma L}$ type appears.

Our model is based on a schematical formulation of rishon model, where the existence of 3-generation structure of cold-sector is naturally explained. This schematical model should be supposed to be concerning to the sub-structure behind the GUTs structure of leptons and quarks. Though our model can explain the 3-generation structure, it can not explain so sufficiently why the mass of top quark is so heavy. It is reduced to badly broken symmetry caused by yet to be known some mechanism. Natural explanation of the large mass of it is further problem. Further, precise decision of neutrino mass and oscillation pattern in the lepton sector will light on the related problems[19].

It should be emphasized that almost all quantum numbers including lepton and quark numbers are not conserved in GUTs. The rishon model is just the one based on the most fundamental electric charge, which is exactly conserved in GUTs. In this sense, the rishon model is very remarkable model. Further, it is probable some of these ‘‘particles’’ in hot-sector generations are the ones beyond

ordinary particle picture. It is not yet known how behave these particles. The problem of the upper bound of flavor number in ordinary field theory should be examined in this context[20].

Finally, if new event concerning to new particles is discovered, we should examine the possibility that it is the one belonging to the hot-sector generations, together with one in GUTs or super-symmetric GUTs.

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