NUMERICAL SOLUTION OF RUNOFF PREDICTION BY WIENER'S THEORY (II)

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1. Introduction

The previous paper could not express perfectly the relationships between natural rainfall and the drainage basin, so the results could not be perfectly predicted. The phenomenon of the runoff process, however, can be better understood and mathematically formulated by introducing Statistics and Information Theories. In order to better predict the amount of runoff from the rainfall, it is necessary to determine the actual input values. It is assumed that the components of any given precipitation consist of two parts: (1) water losses,and (2) the difference between the total percipitation and the water losses.

This is the second report of water useage and management of irrigational reservoirs.

2. Theoretical Background

As shown in Fig.1 in the previous paper[1], we made a linear model. The output of a predictor $y(t)$ may be expressed in the form of a superposition integral,

$$y(t)=\int_{-\infty}^{\infty} w(t_1)x(t-t_1)dt_1 \qquad (1)$$

where t and t_1 are dummy variables, and $x(t)$ and $w(t)$ represent the input and the impulsive response functions of the predictor respectively.

The impulsive response of the predictor is ascertained by

$$\int_{-\infty}^{\infty} w_m(t_2)\phi_{xx}(t_1-t_2)dt_2-\phi_{xz}(t_1)=0 \qquad (2)$$

Eq.(2) is called the Wiener-Hopf equation. Although in Eq.(2) t is assumed to be infinite; In practice it is finite. Therefore, we changed Eq.(2) to matrix form as shown in Eq.(3).

$$\begin{vmatrix} \phi_{xx}(0) & \phi_{xx}(1) & \cdots & \phi_{xx}(m) \\ \phi_{xx}(1) & \phi_{xx}(0) & \cdots & \phi_{xx}(m-1) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{xx}(m) & \phi_{xx}(m-1) & \cdots & \phi_{xx}(0) \end{vmatrix} \begin{vmatrix} W(0) \\ W(1) \\ \vdots \\ W(m) \end{vmatrix} = \begin{vmatrix} \phi_{xz}(0) \\ \phi_{xz}(1) \\ \vdots \\ \phi_{xz}(m) \end{vmatrix} \qquad (3)$$

The linear method for the prediction is obtained by Eqs.(1) and(2).

3. Study Procedures

In this study, it is assumed that precipitation consists of two major components. One is water loss, the other is precipitation excess. Water loss is here defined as the difference between the total precipitation and the total runoff for any given area. It may be subdivided as follows; (1) *interception*, (2) *evaporation from the water surface*, (3) *plant transpiration*, (4) *soil or land evaporation*, *and* (5) *water leakage*.

These different losses can not be easily segregated to permit a separate and independent measurement of each. However, the interrelationship of these losses tends to make their total more nearly constant in any particular region or climate.

Interception; The interception rate is greatest at the beginning of a storm and decreases with the duration of the storm. The total amount of interception increases with the duration of a storm, and since the depth of precipitation increases with duration, there is a general correlation between total interception and total rainfall. There is however little difference between the loss resulting from a heavy downpour and from a light rainfall. It is assumed that topography, soil, condition of vegetation, and other watershed characteristics remain constant through-out the irrigational period (from June to October). The interception storage expressed by

$$I_n(x)=3.31-3.31\times 0.914^x \tag{4}$$

where $I_n(x)$ is the total interception, and x is the total rainfall in millimeters.

Evapo-transpiration; The rate of evapo-transpiration varies depending upon the temperature, sunlight, moisture content, and other atmospheric conditions. Several methods may be used to determine them at any basin. From among several methods, the BLANEY and CRIDDIE method was selected to compute the evapo-transpiration at the basin. The precipitation excess is obtained through the above assumptions.

$$x'(t)=x(t)+\sum_{t_1=0}^{i} ET(t-t_1)+I_n(t) \tag{5}$$

where $x(t)$ is the actual precipitation, and $\sum_{t_1=0}^{i} ET(t-t_1)$ represents the integral depression storage.

The variation of the underground flow is usually small, and although some times there may be watershed leakage which should not be overlooked, in this study we have not considered it.

As this method has not been developed to include the effects of snow accumulation or snowmelt, the extent of the application is limited.

4. Results and Discussions

Since the Miai and Nakato observation points are respectively representative of rainfall and streamflow records in this study area, these two observation points were selected for analysis. Rainfall and streamflow records used in this study were selected from May to October 1968. Fig.1 shows the impulsive response of the optimum predictor computed by using Eq.(3). The optimum time lag shows 32 days. The impulsive response of the optimum predictor shows the values to be 91.1 per cent of the ideal predictor, using Eq.(6)

$$\bar{x}=\bar{y}\int_{-\infty}^{\infty} w_m(t)dt \tag{6}$$

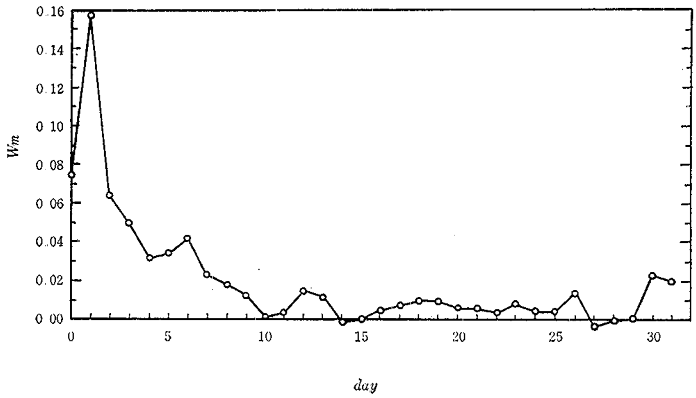

Fig. 1 The impulsive response of the optimum predictor.

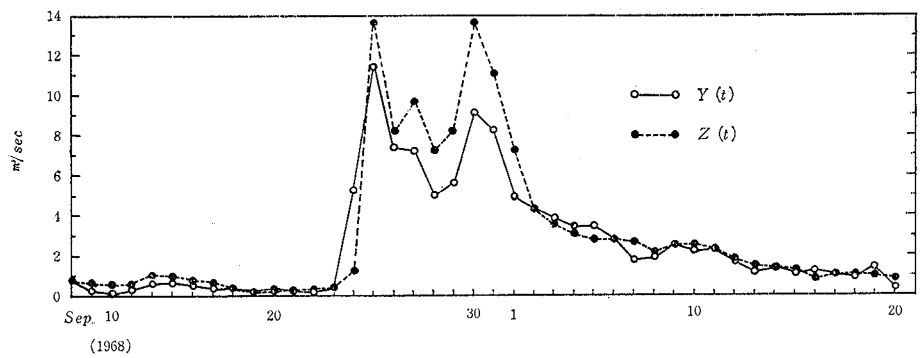

Fig. 2 The comparison of the daily observed and predicted discharge.

where $\bar{x}$ and $\bar{y}$ are the mean precipitation and streamflow respectively. The reason for this may be linear trend, long-period component, and other effects. Fig.2 shows the comparison of the daily observed discharge the predicted discharge. The two curves are much the same. The relative error is between 0.1 and 0.3.

Although the method has some theoretical shortcomings, it has had good results in predicting the runoff from rainfall.

5. Summary

A mathematical model based on the method of storage function can be solved in this way. The degree of accuracy in predicting the runoff from rainfall is not sufficient, partially because the adjustment of this model by the observed data may not be good. Even the assumptions of this linear model have been questioned in recent years, by introducing the Statistics and Information Theories, the problems can be expanded to generalize the functional relationships between the model parameters and the characteristics of the basin and rainfall.

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Wiener の予測理論による流出予測の数値的解法（II）

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要　旨

第1報における線型モデルの予測性は十分ではなかった．そこで線型モデルの入力として損失雨量(Eq. (15))を考えて予測性の向上を試みた．1968年5月1日から1968年11月30日の間の土器川中通観測点について検討を試みた．Fig. 2は最適予測関係を，Fig. 3は観測流出量と予測流出量との比較を示している．図から予測の相対誤差は連続降雨のある場合に少なく，連続旱天日数の多い場合に大きくなることが分かる。また最適予測関係数の最適日数として32日を得たが，その値を積分(Eq. (6))した結果は理想的な予測関数の91.1%を示した．

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