MULTIPLIER ANALYSIS IN THE INPUT-OUTPUT ANALYSIS

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I

The purpose of this paper is to clarify the implications of the "multiplier" in the input-output model. In the process of this review, the input-output model will be examined rather critically in order to explore the conditions that would permit the multiplier analysis to work more effectively for the policy recommendations. Accordingly, this paper should be viewed as the preliminary study for the purposes of the subsequent empirical studies.

The following section II begins with the concept of the multiplier in general. In this section, the simple multiplier is discussed. In the light of this terminology, the more sophisticated multiplier can be derived from the input-output model, which is based on some technical assumptions. These assumptions, therefore, are the subject of section III.

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Section IV digs into the applications of the input-output analysis. The Hirsch method and the Isard-Kuenne method, which represent rather early work in the field, are appraised for the derivation of the multiplier. In section V, the disaggregated basic-service models are compared with the input-output models for the multiregional projections. It also evaluates the location quotients as a tool. The paper closes with concluding remarks and a forward look.

II

Economists have long been interested in measuring the total impact on employment, income, and output resulting from a given change in investment. One of the more useful analytical techniques developed by J. M. Keynes, based on the earlier work of R. S. Kahn, was that of the *multiplier*.

In "The General Theory of Employment, Interest and Money" (1936) Keynes wrote as follows:

> "in a given circumstances a definite ratio, to be called the *Multiplier*, can be established between income and investment and, subject to certain simplifications, between the total employment and the employment directly employed on investment (which we shall call the primary employment)."

After he assessed the multiplier introduced by Kahn, he explained his own concept anew (i.e., the investment multiplier):

> "Our normal psychological law that, when the real income of the community increases or decreases, its consumption will increase or decrease but not so fast, can, therefore, be translated— not, indeed, with absolute accuracy but subject to qualifications which are obvious and can easily be stated in a formally complete fashion—into the propositions that ΔC_w and ΔY_w have the same sign, but $\Delta Y_w > \Delta C_w$, where C_w is the consumption in terms of wage-units.

Let us define, then, dC_w/dY_w as the *marginal propensity to consume*. This quantity is of considerable importance, because it tells us how the next increment of output will have to be divided between consumption and investment. For $\Delta Y_w = \Delta C_w + \Delta I_w$, where ΔC_w and ΔI_w are the increments of consumption and investment: so that we can write $\Delta Y_w = k \cdot \Delta I_w$, where 1-1/k is equal to the marginal propensity to consume.

Let us call k the *investment multiplier*. It tells us that, when there is an increment of aggregate investment, income will increase by an amount which is k times the increment of investment."

Since Keynes, in his theory, dealt in broad aggregates, his income and employment multipliers were also highly aggregated. As a matter of fact, the concept of an aggregated multiplier is, by itself, a useful one. It plays an important role in public policy decisions, insofar as we are interested in the overall impacts.[1)]

However, if we are more interested in the details than in the overall impact, how, then, is this best taken into account? Let us suppose, for example, that a decision has been made to stimulate economic activity by means of investment in public works. There will be an immediate impact on the construction industry, but how will the effects of stepped-up construction activtiy ramify throughout the economy? The impacts on the industries most directly affected can be measured with little difficulty. But when one can recognize the interdependence of economic activities, it is apparent that the impact will not be limited to those industries directly affected. These are the subject matter in this paper, where somewhat more disaggregated multipliers are needed for the study.

1) This concept was used, for example, in determining the size of the tax cut which followed enactment of the Revenue Act of 1964. See W. H. Miernyk, "The Elements of Input-Output Analysis," p.42.

Before proceeding with further analysis, let us explain the multiplier more specifically. For the fundamental way of thinking remains the same throughout the paper, which can be fully absorbed in the concept of the *multiplier*. In a general way, we can define the multiplier as follows:

> *Definition*; the multiplier is the number by which the change in one variable must be multiplied in order to present us the resulting change in another variable.

Hence, the word "multiplier" itself is used for the numerical coefficient showing how much is the increase in one variable resulting from each increase in another one. Then, let us clarify the implications of the multiplier analysis by using the following hypothetical example.[2)]

Assume that I hire unemployed resources to build a $1,000 garage. Then as the first impact, my carpenters and lumber producers will get an extra $1,000 of income. But, that is not the end of the story. If they all have a marginal propensity to consume of 2/3, they will now spend $666.67 on new consumption goods. The producers of these goods will now have an extra income of $666.67. If their MPC is also 2/3, they in turn will spend $444.44, or 2/3 of $666.67 (or, 2/3 of 2/3 of $1,000). So the process will go on, with each new round of spending being 2/3 of the previous round.

Thus, a whole endless chain of secondary consumption respending is set up by my primary $1,000 of investment spending. But, although an endless chain, it is a dwindling chain. By virtue of an infinite geometric progression, it adds up to a finite amount. Diagrammatically, we can show these repercussion effects as follows:

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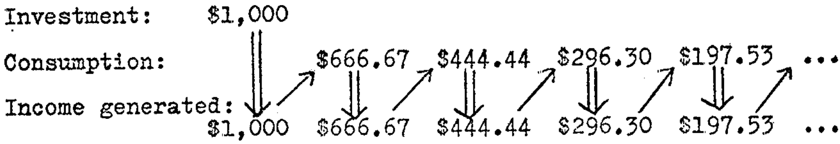
2) This writer owes the following hypothetical example to P. A. Samuelson, "Economics" (8th edition) pp.215-217.

Figure 1. Income Generating Process



Total amount of income generated:

$$
\begin{aligned}
&\$1{,}000+\$666.67+\$444.44+\$296.30+\ldots\ldots \\
&=\$1{,}000+\$(2/3)1{,}000+\$(2/3)^2\,1{,}000 \\
&\quad+\$(2/3)^3\,1{,}000+\ldots\ldots\ldots \\
&=\$\{1/(1-2/3)\}\,1{,}000 \\
&=\$3(1{,}000) \\
&=\$3{,}000.
\end{aligned}
$$

In short, if we let ΔY denote the total amount of income generated, it can be precisely calculated by the following formula:

$$\Delta Y=\{1/(1-\mathrm{MPC})\}\cdot\Delta I$$

where $1/(1-\mathrm{MPC})$ is called the investment multiplier and ΔI is called the multiplicand. It should be noted that the size of the multiplier depends on how large the MPC is. For our example, this shows that with an MPC of 2/3, the multiplier is 3, consisting of the 1 of primary investment plus 2 extra of secondary consumption respending. If the MPC were 3/4, then the multiplier would be 4, and if it were 1/2, the multiplier would be 2, etc., since the simple multiplier is always the reciprocal of the marginal propensity to save (i.e., 1 - MPC). In other words, the greater the extra consumption respending, the greater the multiplier. This is the implication of the "simple multiplier."

Now, let us look into the real world. It by no means looks so simple as that above stated. It has been changing over time as well as over space. For example, the location of new industry in some region always follows the structural change of that area and its vicinity. This is well known as the agglomeration effect. Facing with the situation like this, how can we measure its resulting effects in advance? In this context lies the role of multiplier analysis.

The relevance of multiplier studies for programming regional development is obvious. It neatly points up how growth in one sector induces growth in another. The relevance of such studies for understanding regional cycles is also obvious as soon as we recognize that some impulses may be positive, others negative; some expansionary, others deflationary. Regional multiplier analysis can be designed to handle any number of variables. Yet, the more variables a design encompasses, the more difficult it is to leave the conceptual stage and derive results of direct usefulness.[3)]

In the following section, we will discuss the input-output model and examine the underlying assumptions critically. For the most comprehensive regional multiplier analysis seems to be much associated with the use of the input-output technique.[4)]

III

The input-output model (or, the analysis of interindustrial relations) is known as one of the central subjects in the field of modern economics. Historically speaking, in 1931, W. W. Leontief got started on the work of completing the U. S. Input-Output Table by himself.[5)] The reason why the input-output analysis was so keenly attracted the economists' attentions may be attributable to the fact that it could succeed in correct prediction for the U. S. economy immediately after the last

3) See W. Isard, "Methods of Regional Analysis," p.189, pp.194-205.

4) As regards the economic impacts, A. O. Hirschman made a distinction between the "backward linkage effect" and "forward linkage effect." For the detailed discussion, see A. O. Hirschman, "The Strategy of Economic Development," pp.100-104. He also briefly referred to its relations with the input-output model.

5) Leontief's basic ideas were first published in his article, "Quantitative Input-Output Relations in the Economic System of the United States," the Review of Economics and Statistics, XVIII, (1936).

world war II. In this sense, to predict or forecast the structure of the economy was the main purpose of the input-output analysis at least in the beginning.

However, it seems to me that the purpose of the studies based on the input-output model has been gradually changing. Leontief has noticed this trend and stated, in the preface to "Structural Interdependence and Economic Development," as follows:

> "The first international conference dealt largely with the empirical implementaion of input-output systems. The major emphasis of the second conference was on statistical and computational procedures and problems. The central theme of the third conference was the application of the input-output analysis to projection and developmental planning."

Thus, during the decade spanning the three international conferences, there was a marked shift from emphasis on the problems of constructing input-output systems to the application of these systems to a variety of economic problems.[6]

In short, the historical trend in the input-output studies can be characterized as the gradual shift from the fundamental constructions of the input-output tables to its applications to a variety of economic problems. It should be emphasized that in the light of the historical trend above stated, the underlying assumptions of the input-output model should be carefully scrutinized, when we apply it to some particular problems such as the impact analysis under study. The input-output analysis has had and continues to have its critics. This is, however, not at all unusual. Indeed, it would be unfortunate if the situations were otherwise. The advancement of knowledge is accelerated by constructive, scientific criticism. Weaknesses in any system of thought can be better

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6) See W. H. Miernyk, "The Elements of Input-Output Analysis," p.80.

attacked if they are pinpointed by detailed critical analysis. This is true not only of input-output analysis but of any scientific endeavor, whether in the physical or the social sciences.[7]

Let us review the underlying assumptions of the input-output model, and ask ourselves again why they are introduced into the model explicitly? As regards the recognition of the real economic structure, Leontief finds it as follows:

> "the real world requires you to recognize the whirlpools of industrial relationships characteristic of general models of interdependence. For the production of coal, iron is required; for the production of iron, coal is required; no man can say whether the coal industry or the iron industry is earlier or later in the hierarchy of production."[8]

Therefore, insofar as we admit the Leontief's recognition to be plausible or reasonable, we are allowed to build up the input-output table, since by using it we can expect to catch up the round-by-round process of interactions among regions as well as industries. In other words, we can derive the comprehensive (or, disaggregated) multipliers from the ordinary input-output table.

The input-output analysis is based on the following three tables:

1) the transaction matrix table,
2) the input coefficient matrix table,
3) the inverse matrix table.

Among them, the first table is the most important. It has a property of double-entry system where every cell stands for an input as well as an output. Owing to this property, we come to obtain a clear idea of

7) Ibid., p.126.
8) Dorfman, Samuelson and Solow, "Linear Programming and Economic Analysis," Chapt. 9, (1958).

the structural characteristics of one industry compared with the others.[9)]

Suppose an economy divided into two endogenous sectors and one exogenous sector. Then, the associated transaction matrix table can be written as follows:

Table 1. Transaction Matrix Table

	I	II	Final Demand	Gross Output
I	x_{11}	x_{12}	F_1	X_1
II	x_{21}	x_{22}	F_2	X_2
V. A.	V_{01}	V_{02}	—	V_0

where

x_{ij} = sales by sector i to sector j,

F_i = sales by sector i to exogenous demand,

$X_i = \sum_j x_{ij} + F_i$ = gross output of sector i,

V_{0j} = purchases by sector j from exogenous sector,

V_0 = total value-added in the economy.

It should be kept in our minds that the above given transaction matrix table should be viewed as the *descriptive device* to the effect that it can depict the structure of the real economy in a systematic way. Furthermore, if we want to make use of the transaction table not only as a mere descriptive device, but also as an *analytical tool*, we then have to set up some strong technical assumptions in it. These assumptions are summarized as follows:

1) constant returns to scale,
2) convexity of the isoquant surfaces,
3) fixed coefficients of production.

If we admit all of these assumptions, then the following two

9) For example, see H. Yamada & T. Ihara, "An Interindustrial Analysis of the Transportation Sector," the Kyoto University Economic Review, (1969).

tables (i.e., the input coefficient matrix table and the inverse matrix table) can be readily calculated to serve as the efficient tools in a variety of economic problems. Table 2 shows the input coefficient matrix table which would be derived from Table 1:

Table 2. Input Coefficient Table

	I	II
I	a_{11}	a_{12}
II	a_{21}	a_{22}
V.A.	a_{01}	a_{02}

where a_{ij} $(i=0,1,2;\ j=1,2)$
$=x_{ij}/X_j$ for $i=1,2;\ j=1,2,$
$=V_{0j}/X_j$ for $i=0\ ;\ j\ =1,2.$

An input coefficient, a_{ij}, means the amount of inputs required from each sector i to produce one dollar's worth of output of a given sector j.

A more integral part of input-output analysis is the construction of an inverse matrix table, which shows the direct and indirect effects of unit change in final demands. Specifically, it shows the total expansion of output in all sectors as a result of the delivery of one dollar's worth of output outside the endogenous sectors by each sector. A delivery outside the endogenous sector means a sale to households, or any other buyer included in the final demand sector. As Table 3, the associated inverse matrix table is expressed:

Table 3. Inverse Matrix Table

	I	II
I	b_{11}	b_{12}
II	b_{21}	b_{22}

where b_{ij} $(i,j=1,2)$ is the associated element in the Leontief inverse matrix, i.e., $(I-A)^{-1}$,

Mathematically, it can also be expressed in terms of the power series as

$$(I-A)^{-1}=I+A+A^2+A^3+A^4+A^5+\ldots\ldots\ldots\ldots\ldots$$

In short, this table shows the total requirements, direct and indirect, per dollar of delivery outside the endogenous sector.

So far we have seen that the introduction of the technical assumptions into the transaction table made it possible for us to derive the tables of the input coefficient matrix and its inverse with little difficulty. Mainly used for our analysis are the latter two tables. Therefore, before obtaining some significant conclusions based on these tables, we must carefully examine the implications of the above stated technical assumptions.

The first assumption (i.e., the constant returns to scale) signifies that the production function has the property of homogeneity of degree one. As for our simple example, the Leontief production function can be written as follows: let a_{ij} be the required minimal input of commodity i per unit of output of commodity j, where $i=0,1,2$; $j=1,2$. Then,

$$X_1 = Min\ (x_{11}/a_{11},\ x_{21}/a_{21},\ V_{01}/a_{01}),$$
$$X_2 = Min\ (x_{12}/a_{12},\ x_{22}/a_{22},\ V_{02}/a_{02}).$$

If each input x_{ij} is increased by the factor "t", output is also increased by the factor "t". Hence, the property of constant returns to scale is readily verified.

The second assumption (i.e., the convexity of the isoquant surfaces) signifies the generalized diminishing returns. However, in the input-output model, all the isoquant surfaces are assumed to have the right-angled corner, and thereby meet the second condition as a special case. Viewed differently, it means that the elasticity of factor substitution is always zero.[10] If we let σ denote the elasticity of substitution, and MP_{1j}, MP_{2j} denote the marginal productivity of production factor 1 and 2, respectively, then we have

10) For the detailed discussions on the elasticity of substitution, see Arrow, Chenery, Minhas, and Solow, "Capital-Labor Substitution and Economic Efficiency," the Review of Economics and Statistics, (1961), pp. 228—232.

$$\sigma=\frac{d\ log(x_{2j}/x_{1j})}{d\ log(MP_{1j}/MP_{2j})}=\frac{MP_{1j}/MP_{2j}}{x_{2j}/x_{1j}}\cdot\frac{d\ (x_{2j}/x_{1j})}{d\ (MP_{1j}/MP_{2j})}=0.$$

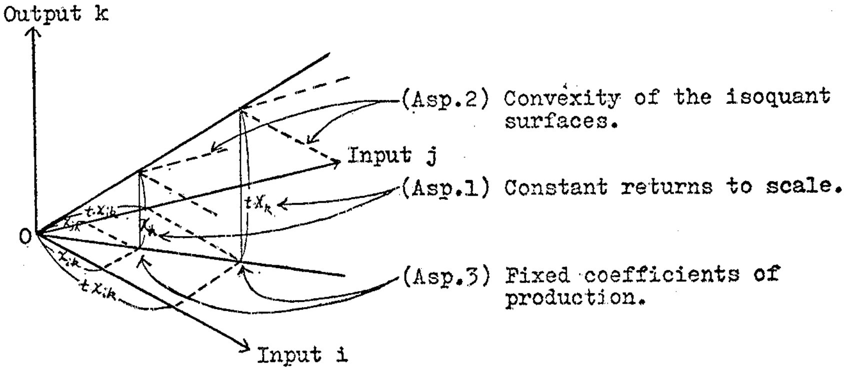
The fixed coefficients of production belong to the third assumption. In appraising the input-output model, we need not discuss the qualifications springing from the use of constant production coefficients. They have been fully discussed elsewhere.[11] However, the discriminating of the varied viewpoints requires some discussions. Insofar as the evaluation of constant production coefficients, they may be classified into the following three broad categories:

1) In the light of the theoretical as well as empirical materials examined so far, this assumption is very dubious. Hence, it may be permissible only as the first approximation for our analysis.
2) Apart from the theoretical examination, it may be permissible simply because it has been fully verified by the empirical and statistical tests.
3) Not only from the empirical and statistical viewpoints, but also from the theoretical viewpoints under certain circumstances, the fixed coefficients of production may be fully permissible.

The order of arrangement of the above statements is followed by the grade of how much they are in favor of the given assumption. Accordingly, the third viewpoint, which is based on the so-called "substitution theorem",[12] seems to be most advocating the constant production coefficients. Diagrammatically, all of the technical assumptions for the input-output model are pictured in Figure 2.

11) See W. Isard, "Methods of Regional Analysis," pp. 338—343.
12) As regards the theoretical verifications of the "substitution theorem", see T. C. Koopmans, ed., "Activity Analysis of Production and Allocation," (1951).

Figure 2. Technical Assumptions in the Input-Output Model



In short, the foregoing discussion can be summarized as follows. Any model is always based on some assumptions. If the conclusions derived from the model turned out to be misleading, then the underlying assumptions of its model should be fully scrutinized above all things. We have so many input-output applications to a variety of economic problems, but all of them are basically stemmed from these assumptions. It should be noted that these assumptions, such as the fixed coefficients of production, the elasticity of factor substitution being zero, and the constant returns to scale, are introduced only because we want to use the transaction table not as a mere descriptive device but as an analytical tool for our study. Hence, with an aid of these assumptions, we are able to analyze the real world more effectively and more meaningly. The merits of the input-output analysis lies in the systematic operationality.

IV

All of the interregional input-output tables constructed in the United States to date and all of the early regional tables were based on input coefficients taken from the national table. The procedure in constructing such tables was to estimate total gross output figures for each industry and sector in the region or regions to be analyzed. These figures, for each industry and sector, were then multiplied by *national* input coefficients.[13]

The result in each case was a table of interindustry flows based on the assumption that *regional input patterns were identical to national input patterns*. This assumption imposes a severe limitation on the use of such input-output tables for analytical purposes. The major problem involved in using national input coefficients to construct regional tables is that of variations in "industry-mix" and "production-mix" from region to region. This problem is minimized if a table of national coefficients is available in great detail. However, even in this case, it is not completely solved. The problem is essentially one of industrial classification.

An important forward step in regional input-output analysis was taken by Moore and Petersen when they constructed their input-output table for Utah. These authors followed Isard's procedure in estimating total gross output figures for the 26 sectors of their transactions table from published sources. Their next step was to use national input coefficients to determine interindustry flows as a first approximation. Following this, the row and column distribution for each sector was modified in the light of differences in regional productive processes, marketing practices, or product-mix. These modifications were based on all the information they could obtain about individual industries, on technical data, and on estimates constructed from employment and income data. Such modifications of national input coefficients were feasible in the Utah study,[14] but they could have been used only at great expense in earlier studies covering larger and more densely popu-

13) In this section, we draw heavily on W. H. Miernyk, Ibid., pp.66—69.

14) For the detaild discussions on the method of Moore-Petersen, see F. T. Moore and J. W. Petersen, "Regional Analysis: An Interindustry Model of Utah," the Review of Economics and Statistics, vol.37, (1955).

lated geographic areas. The Moore-Petersen study served as a model for other regional scientists, however, and marked a major step forward in regional input-output analysis.

1. The Hirsch Method

The next major advance in implementing the *regional input-output model* was made by W. Z. Hirsch in his study of the St. Louis Metropolitan area. Hirsch followed the customary practice of obtaining gross output figures, and other "control totals" from published sources. He did not, however, apply national coefficients to these control totals to obtain interindustry flows. Instead, input and output data were obtained for most large and medium sized companies operating in the St. Louis area...each of these companies assigned one of its key officials to work with the research staff of this study for a three-months' period. Each company prepared its own input-output table for 1955. The participants in the study were carefully briefed orally and given written instructions to ensure uniformity of reporting. Where only a sample of firms in an industry was included in the survey, the sample results were "blown up" on the basis of employment data. Once the interindustry flows had been established, the aggregated results could be compared with control totals obtained from published data, and the necessary reconciliations were made.[15)]

While the method employed by Hirsch is expensive and time-consuming, there is little doubt about its superiority to other estimating techniques. Because, one of the major criticisms of regional input-output analysis, made before Hirsch published the results of his study, was that of using national coefficients at the regional level. By using primary data Hirsch avoided this criticism. But it must be emphasized that the more accurate input coefficients derived from the St. Louis table were ob-

15) See W. Z. Hirsch, "Interindustry Relations of a Metropolitan Area," the Review of Economics and Statistics, vol. 41, (1959).

tained only at relatively ***high cost***.

Since publication of the Moore-Petersen and Hirsch studies, few regional input-output analyses have relied on national input coefficients. For one thing, by the late 1950's it was recognized that the 1947 national input coefficients could no longer be used without major adjustments. Some of the more recent studies haved used the Moore-Petersen approach of applying adjusted national coefficients to state or regional control totals. Others, however, have followed Hirsch's lead in conducting surveys to obtain estimates of interindustry flows.

Based on this interindustry flow table, Hirsch attempted some interesting activity and impact projections. We now turn to a discussion of the *sectoral multipliers* derived from his study of the St. Louis metropolitan area. For these concepts may be viewed as powerful tools in assessing the impact of final demand changes on the economic activity of a metropolitan area.

The first step in the development of sectoral multipliers is to close the basio transactions table with respect to households, if we are interested in the local multiplier effect resulting from new household income generation in the area. The next step is to compute the direct and indirect requirements per dollar of final demand for the new system which includes households in the processing (or, endogenous) sector. To evaluate and compare the income impact of final demand changes, two kinds of income multipliers were calculated by Hirsch. The multipliers and the details of his calculation are given in Table 4.

The first column in Table 4 is the household row of the input coefficient table including households in endogenous sector (let this matrix be A_h). Column 2 is the sum of the household coefficient of the corresponding column of A_h times each column entry of the inverse matrix which has not included households in endogenous sector (let this

Table 4. Income Interactions in the St. Louis Metropolitan Area, 1955

	Industrial Sector	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1.	Food and kindred products	.14	.23	.09	1.77	.36	.13	.22	2.57
2.	Textiles and apparel	.32	.41	.09	1.28	.64	.23	.32	2.00
3.	Lumber and furniture	.35	.49	.14	1.41	.77	.28	.42	2.20
4.	Paper and allied products	.26	.39	.13	1.50	.62	.23	.36	2.38
5.	Printing and publishing	.45	.56	.11	1.24	.87	.31	.42	1.93
6.	Chemicals	.25	.32	.07	1.28	.51	.19	.26	2.04
7.	Products of petroleum & coal	.08	.13	.05	1.72	.22	.09	.14	2.75
8.	Leather and leather products	.38	.47	.09	1.25	.75	.28	.37	1.97
9.	Iron and steel	.35	.46	.11	1.30	.73	.27	.38	2.08
10.	Nonferrous metals	.27	.40	.13	1.51	.64	.24	.37	2.37
11.	Plumbing & heating supplies; fabricated structural metal products	.36	.45	.09	1.27	.71	.26	.35	1.97
12.	Machinery (except electrical)	.31	.44	.13	1.44	.70	.26	.39	2.26
13.	Motors & generators; radios; other electric machinery	.44	.53	.09	1.22	.84	.31	.40	1.91
14.	Motor vehicles	.17	.28	.11	1.72	.45	.17	.28	2.65
15.	Other transportation equipment	.33	.37	.04	1.13	.59	.22	.26	1.79
16.	Miscellaneous manufacturing	.37	.53	.16	1.43	.85	.32	.48	2.30
17.	Coal; gas; electric power; water	.26	.35	.09	1.35	.58	.23	.32	2.23
18.	Railroad transportation	.39	.51	.12	1.29	.81	.30	.42	2.08
19.	Other transportation	.43	.54	.11	1.25	.86	.32	.43	2.00
20.	Trade	.61	.73	.12	1.19	1.16	.43	.55	1.90
21.	Communications	.44	.49	.05	1.10	.79	.30	.35	1.80
22.	Finance & insurance; rentals	.34	.50	.16	1.48	.84	.34	.50	2.47
23.	Business and personal services	.57	.74	.17	1.29	1.16	.42	.59	2.03
24.	Medical, educational & non-profit organ.	.77	.86	.09	1.11	1.34	.48	.57	1.74
25.	Undistributed	.36	.49	.13	1.36	.82	.33	.46	2.28
26.	Eating and drinking places	.35	.51	.16	1.48	.82	.31	.47	2.34
27.	Capital construction & maintenance.	.40	.59	.19	1.47	.93	.34	.53	2.32

(1) Direct income change
(2) Direct & indirect income change
(3) Indirect income change
(4) Multiplier (Model I)
(5) Direct, indirect & induced income change
(6) Induced income change
(7) Indirect & induced income change
(8) Multiplier (Model II)

Source: W. Z. Hirsch, Ibid., p.365.

inverse matrix be $(I-A)^{-1}$).
In column 3, the difference between column 2 and column 1 is written. From these numerical results, the first multiplier, what he calls *model 1*, is derived, and is presented in column 4, which is equal to column 2 divided by column 1.

It is evident that the first type of multiplier may be referred to as a "simple income multiplier," since it takes into account only the direct and indirect changes in income resulting from an increase of one dollar in the output of all the industries in the processing sectors. In other words, it assumes that neither consumer expenditures nor investment expenditures for new plant and equipment are affected. In order to make allowance for consumer expenditure adjustments, he steps forward to yield the second type of multiplier, what he calls *model 2*. Column 5 is the household row of the inverse matrix which includes households in endogenous sector (i.e., the household row in $(I-A_h)^{-1}$). For the further reference, column 5 minus column 2 is written in column 6, and then column 3 plus column 6 is written in column 7. Finally in column 8, the second type of multiplier, which is equal to column 5 divided by column 1, is presented. It should be noted that the second type of multiplier is a more realistic measure which takes into account the direct and indirect effects indicated by the input-output model plus the *induced changes in income* resulting from increased consumer spending. In this sense, for each sector, the second type of multiplier will always be larger than its first type counterpart.

Now the *implications* of the above defined multipliers are elucidated. They reveal that different amounts of income are generated by different sectors of the economy even if we assume that each sector expands its output by the same amount.[16] The first type of multipliers

16) As for differences in the direct income effect, Hirsch points out the following four main reasons: i) the relative differences in wage level, ii) the

are limited to the direct and indirect effects on income of a given change in output, but the second type of multipliers also show the chain reaction of interindustry reactions in income, output, and once more on consumer expenditures. It is clearly the interindustry flow model that makes possible the tracing and evaluation of this *chain reaction*.

There are some technical problems involved in computing income multipliers. The most empirical input-output multipliers have been local or regional, and among the problems involved in conducting regional input-output analyses are those resulting from the lack of data on consumer spending patterns for small areas. Hence, we need some additional assumptions about consumer behavior. Hirsch, for example, assumed that "changes in consumer spending were proportional to changes in income." Because of this assumption, he had overstated the income effects of changes in final demand. However, this should *not* be taken for a criticism of the Hirsch studies, since he is fully aware of the limitations of his consumer data, and specifically points out the effects which his assumption had on the regional multipliers he computed.[17]

In short, we may conclude with the following statements. One should not exaggerate the limitations of sectoral multipliers computed from regional input-output models because of the underlying assumptions about consumer behavior. One assumption, if it were not plausible, can well be substituted for another one. By so doing, we come to reach far better assumptions. For many analytical purposes, they are more useful and revealing than aggregate multipliers which relate only to the economy as a whole.

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differences in labor intensity, iii) the differences in labor productivity, iv) the relative importance of imported inputs, as the most decisive factors. See W. Hirsch, op. cit., pp.363—364.

17) Hirsch has obtained some interesting fact-findings concerning the relationships among the varied multipliers, which will be discussed and statistically tested below in Appendix.

2. The Isard-Kuenne Method

Attention is now turned to the method for computing *employment multipliers* used by Isard and Kuenne to project estimated total employment in the Greater New York-Philadelphia region as a result of the expansion of the steel industry in the area. From the methodological point of view, their studies are related to the iterative technique for obtaining estimates of the direct and indirect requirements per dollar of sales to final demand.[18)]

They state as follows: [19)]

> "It is the purpose of this paper to attempt some development in the theory of agglomeration and of broad spatial clustering of all economic activities by grafting on to the sounder elements of location theory a modified regional input-output schema.
>
> Viewed from a different standpoint, we attempt an impact study, wherein the direct and indirect repercussions of the location of a basic industry in a region are evaluated."

In this context, the first step in the Isard-Kuenne analysis was to estimate the agglomeration effect by analyzing the clustering of establishments around a similar basic installation in other area. The next step was to estimate the shifts in production that would occur in the region under study. Following this, estimates of production-worker employment were made for each of the "satellite industries which were expected to be attracted to the new basic industry. Up to this point, the analysis depended heavily upon location theory and informed judgement.

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18) On the derivation of the inverse matrix, there exist two methods, which might be called the direct (or elimination) method and the indirect (or iterative) method, respectively. The latter has at least two advantages: i) computational procedures are remarkably simple, ii) economic interpretations can readily be given.

19) See W. Isard & R.E. Kuenne, "The Impact of Steel upon the Greater New York-Philadelphia Industrial Region," the Review of Economics and Statistics, vol.35, (1953).

The next step was to estimate the "bill of goods" which would have to be furnished to the area. This consisted of all inputs which would be absorbed by the basic industry plus the satellite industries which would be attracted to it by the agglomeration effect. This is the point at which *input-output analysis* was introduced into the study. With reference to this point, they state as follows:

> "In designing our study we made an important modification of the traditional input-output matrix. Since in regional analysis it is important to catch the local multiplier effect resulting from the generation of new income, households have been introduced into the structural matrix as an industry. Hence, the labor and other household services required by the new activities are recorded."

Each of the coefficients in this table was multiplied by the dollar volume of its expected production derived from the employment estimates. This was done for both the basic and the satellite activities to obtain the *total* initial input requirements. Following this, the *minimum* input requirements to be produced in the area were estimated. The figures were derived by Isard-Kuenne by again relying on location theory and informed judgement.

After all the estimates had been made, a table was constructed listing the basic industry and all other industries in the area. It is reproduced as Table 5 below. The totals for each 45 inputs are recorded in column 1 of the Table. After the bill of goods had been constructed, it was necessary to determine the *fraction* of the total amount of each input minimally expected to originate in the area. This step was necessary, since in their study the expansionary effect on the economy of the area is maintained by only the outputs which are produced within the area.

The determination of the above fraction for each industrial activity was based on the small amount of location theory directly relevant, on

Table 5. Direct and Indirect Repercussion of

	Industrial Sector	(1)	(2)	(3)	(4)
1.	Agriculture & fisheries	50.0	0	0.0	0
2.	Food & kindred products	294.6	60	176.8	17,660
3.	Tobacco manufactures	0.0	0	0.0	0
4.	Textile mill products	3,864.7	10	386.5	406
5.	Apparel	1,285.6	75	964.2	10,124
6.	Lumber & wood products	5,610.7	5	280.5	93
7.	Furniture & fixtures	1,753.4	33	578.6	802
8.	Paper & allied products	4,818.7	40	1,927.5	1,674
9.	Printing & publishing	425.5	90	383.0	5,929
10.	Chemicals	10,626.4	45	4,781.9	3,599
11.	Products of petroleum & coal	10,936.6	25	2,734.2	2,547
12.	Rubber products	8,381.5	15	1,257.2	355
13.	Leather & leather products	647.7	20	129.5	679
14.	Stone, clay, & glass products	9,031.7	15	1,354.8	441
15.	Iron & steel	121,170.5	50	60,585.3	13,566
16.	Nonferrous metals	33,997.4	20	6,799.5	1,667
17.	Plumbing & heating supplies	3,192.4	25	798.1	248
18.	Fabricated structural metal prod.	3,480.7	40	1,392.3	312
19.	Other fabricated metal products	31,770.9	40	12,708.4	2,146
20.	Agric'l, mining, & const. machinery	3,651.3	5	182.6	46
21.	Metal-working machinery	7,389.1	25	1,847.3	270
22.	Other machinery (except electric)	28,463.6	40	11,385.4	2,675
23.	Motors & generators	11,265.9	20	2,253.2	226
24.	Radios	4,562.2	30	1,368.7	428
25.	Other electrical machinery	21,773.9	50	10,887.0	2,011
26.	Motor vehicles	50,530.8	10	5,053.1	742
27.	Other transportation equipment	2,605.5	20	521.1	276
28.	Professional & scientific equip.	3,221.4	50	1,610.7	801
29.	Miscellaneous manufacturing	5,116.8	60	3,070.1	2,888
30.	Coal, gas, & electric power	7,767.0	50	3,883.5	1,843
31.	Railroad transportation	13,575.8	75	10,181.9	6,010
32.	Ocean transportation	457.3	75	343.0	331
33.	Other transportation	4,179.4	95	3,970.4	8,422
34.	Trade	13,969.8	95	13,271.3	36,585
35.	Communications	1,790.7	90	1,611.6	2,409
36.	Finance & insurance	3,086.2	90	2,777.6	9,472
37.	Rental	3,018.8	95	2,867.9	26,222
38.	Business services	5,338.5	95	5,071.6	2,385
39.	Personal & repair services	396.9	95	377.1	14,399
40.	Medical, educ., & nonprofit org's	0.0	90	0.0	9,811
41.	Amusements	0.0	90	0.0	3,677
42.	Scrap & miscellaneous industries	8,388.2	50	4,194.1	2,054
43.	Undistributed	103,638.6	50	51,819.3	5,875
44.	Eating and drinking places	0.0	95	0.0	16,916
45.	Households	348,281.0	82	285,590.4	63,002
	Totals	903,807.7		521,377.2	282,024

Source: W. Isard and R. E. Kuenne Ibid., P.299.

New Basic Steel Capacity

(5)	(6)	(7)	(8)	(9)
0	0	0		0
8,249	42,492	1,833		1,833
0	0	0		0
39	1,280	142		142
3,461	21,155	2,302		2,302
36	450	64		64
198	2,000	234		234
1,297	6,574	426		426
3,014	14,617	1,667		1,667
1,630	12,077	601		601
1,118	7,634	228		228
102	1,879	169		169
194	1,371	150		150
139	2,083	268		268
2,965	78,335	6,093	11,666	17,759
381	9,063	505		505
50	1,189	118	3,640	3,758
33	1,809	151	1,420	1,571
561	16,121	1,537	10,060	11,597
11	251	22	707	729
43	2,210	289	2,705	2,994
551	15,384	1,486	28,607	30,093
42	2,560	301		
101	2,026	192	10,392	12,312
432	13,903	1,427		
260	6,421	389	8,770	9,159
69	958	117	4,605	4,722
287	3,123	416		416
982	8,418	845	6,108	6,953
2,693	11,079	1,100		1,100
2,390	21,532	3,308		3,308
170	1,021	110		110
2,836	19,694	2,394		2,394
11,855	83,642	13,874		13,874
1,283	7,305	1,191		1,191
5,062	25,252	2,329		2,329
9,603	55,680	909		909
2,406	13,384	1,305		1,305
5,088	24,212	4,443		4,443
2,160	17,271	4,370		4,370
1,066	6,591	1,100		1,100
727	7,411	771		771
6,019	69,236	7,208		7,208
3,903	29,551	3,705		3,705
80,894	509,578			
164,400	1,177,822	70,089	88,680	158,769

(1)
Input requirements of inital steel and steel-fabricating activities (*in $ thousand*)

(2)
Minimum percentage of input requirements to be produced in area

(3)
First round expansions in area(*in $ thousand*)

(4)
Second round expansions in area(*in $ thousand*)

(5)
Third round expansions in area(*in $ thousand*)

(6)
Sum of round expansions in area(*in $ thousand*)

(7)
Total new employees corresponding to round expansions

(8)
Total new employees in initial steel and steel-fabricating activities

(9)
Over-all total of new employees

the accumulated production and derived consumption data for their area, on data of flows of commodities into and out of the area, and finally, on judgements of informed persons within the area. These fractions are recorded in percentage terms in column 2 of Table 5. It should be noted that again, many *subjective decisions* had to be made, and in each case they attributed to the area the production of only that fraction of the inputs required of an activity, for which they felt there was a firm basis.

Applying the percentages of column 2 to the corresponding items of column 1 yields column 3, which records (as firm minimums) the first round expansion of each industrial activity in the area which they anticipate will be required to maintain the new steel capacity and its associated fabricators in reasonably full operation. It should be noted that in column 3 the requirements of steel and of the diverse outputs of steel-fabricators are considered to represent demand which is over and above the demands for steel and fabricated steel products which initially justified the erection of new basic steel and steel-fabricating capacities. In effect, these industries must produce beyond their initial capacity.

The *employment multiplier* was then derived by computing a series of "rounds of expansion." The first round was computed by applying the percentage of input requirements to be produced in the area to the total input requirements. This procedure was applied successively until several rounds had been computed. After each of the rounds had been computed, they were added together to obtain the sum of round expansions. For each industrial category (except the household sector), the value of output per worker in 1947 was used to obtain a crude estimate of new employment corresponding to the sum of the round expansions of column 6.

Their estimates may be summarized as follows. The new steel mill would employ about 11,700 workers. The agglomeration effect was expected to attract metal-fabricating establishments which would employ additional 77,000 workers. Thus, altogether estimated 88,700 new jobs were expected in the area as a *direct* result of the new steel mill. But on the basis of their employment multipliers, Isard and Kuenne estimated that additional 70,000 new jobs would open up in the area due to the *indirect* effects of the expansion of the basic steel industry in the region. Thus, the estimated *total* employment impact on the area amounted to about 158,700 new jobs.

In appraising the Isard-Kuenne method, some qualifications may be easily pointed out. For example, it was primarily based on the technical assumptions underlying the input-output model, such as the fixed production coefficients. Secondarily, they included the constant consumption coefficients in the structural matrix. Thirdly, they neglected the interregional feedback effects, because of the inability to apply an interregional model at the time of their analysis.[20] These statements, however, do not constitute a criticism of their studies. Specifically, they state as follows:

> "Our effort to achieve an improved agglomeration and regional development analysis has not involved pure theory alone. Rather it has involved a weaving together of diverse theoretical threads and existing empirical material in order to obtain a realistic fabric of an induced development."

Clearly the Isard-Kuenne method was devised to measure the total employment impact on a region resulting from the location of a new basic

20) As regards the interregional feedback effects, the theoretical model as well as some empirical studies are given in H. Yamada & T. Ihara, "Input-Output Analysis of Interregional Repercussion," Papers and Proceedings of the Third Far East Conference of the Regional Science Association, vol. 3, (1963). Note that the Miller's experimental results revealed the fact that the interregional feedbacks were very small and also insensitive to variations in the amount of interdependence between two regions.

industry in that area. Hence, only because of the underlying assumptions in their studies, we should not criticize the concept of *sectoral multipliers* derived from input-output model. In spite of all the basic qualifications, we must recognize that their method still has significance and usefulness as firm minimum projections. What we have to do next is to continue our research of how to obtain (or estimate) the reliable data so as to develop more valid techniques for *projection under uncertainty*.

V

The previous section considers a multiplier in its input-output settings. In general, there are several accepted procedures and techniques of the *multiplier analysis*. They are summarized as the following three basic categories, each with many variations:[21]

1) economic base-type analyses,
2) econometric multiplier models,
3) regional input-output techniques.

Among them, the most comprehensive multiplier analysis is that associated with the use of the *input-output technique*, which we explained in section IV. In contrast, the most simple and straightforward type of regional multiplier analysis is associated with *economic base studies*, which for the most part avoid the interregional variable and employ a very gross industrial classification. The evaluation of each procedure may certainly be affected by the analytical purposes of the studies. Therefore, when we compare the input-output technique with some older issues concerning location quotients and economic base multipliers, an analytical

21) Isard and Czamanski made a comparison of multipliers derived by the several techniques and have shown empirical similarity in the aggregate multipliers derived from economic base models and input-output models. See W. Isard and S. Czamanski, "Techniques for Estimating Local and Regional Multiplier Effects of Changes in the Level of Major Governmental Programs," Peace Research Society (International) Papers, vol. 3, (1965).

purpose, as a common basis, should be clarified for evaluation.

Let us confine ourselves to the discussion concerned not with the descriptive aspects but with the use of the multiplier concepts as ***planning or policy measures***, since the use of a multipier may be viewed as an attempt at prediction. The most of the economic base models, around which an extensive literature has grown, have been widely used for purposes of forecasting long-run urban and regional growth. It has seen fewer applications for purposes of determinig the impact on the local economy of an important development, such as the grant-in-aid programs, a cutback in military expenditures, etc. . Some analysts have made extensive use of the employment multiplier concept for projection purposes. Other analysts have been more cautious about employing the multiplier concept.

One pure theoretical form of the economic base model separates all economic activity in the region into two types, basic and non-basic (or service). The basic activities are assumed to be responsible for building up the regional economy, while non-basic activities exist to serve basic industries and consumers. One formulation of the model assumes that the ratio of total to basic activity is constant.[22] Using E_t=total employment and E_b=employment in basic functions, the ***multiplier*** is then expressed as $M=E_t/E_b$. For any increase in basic activity, total activity is assumed to increase by "M" times the initial change in basic activity.

Whether the basic-service ratio (already designated in the literature by several different terms) and the associated simple regional multiplier are employed for description alone, or are adapted for prediction purposes, numerous limitations are involved in their use. Among these are:[23]

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22) This is identical in results to assuming the ratio of basic to non-basic activity constant or the ratio of non-basic to total activity constant.

23) The many deficiencies of the economic base model have been detailed by Isard and Czamanski, from which the followings are derived. See W. Isard

1) The classification of each activity as either wholly export or wholly service.
2) Alternatively, the division of any particular activity into that fraction which is either export or service.
3) The practical application of the assumption that not only consumption patterns, but also production patterns, are identical when different areas or regions are compared.
4) The failure to recognize and incorporate into the model imports as the counterpart of exports.
5) The failure to incorporate interregional transfers of funds without a corresponding flow of goods.
6) The dependence of the results on the particular industrial breakdown used.
7) Use of the average or highly aggregated multiplier for measuring widely differing phenomena.
8) The problem of differentiating the interindustry effects from other multiplier effects.

Judging from those shortcomings and problems in economic base study, it is quite evident that a regional multiplier derived from the basic-service ratio has a strictly limited degree of usefulness and validity.[24) As an instrument for projection, it can be used only under certain ideal conditions. Even then, it can give no more than an *average or approximate value*. This is not to deny that the economic base study itself is useful. Its value, particularly in a static, descriptive sense, has been and continue to be fully appreciated. The analysts, however, should realize its limitations and should be especially cautious about extending its application to include the computation of regional multipliers for projection purposes.

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and S. Czamanski, op. cit., p.21. Note that they are both technical and conceptual.

24) See W. Isard, "Methods of Regional Analysis," pp.204-205.

Above all, he should supplement its use with other forms and types of regional analysis.

It has been clarified that the economic base approach always concerns how to identify basic and service components. Clearly it is one of the most bothersome technical problems when we must face in constructing basic-service ratios with respect to the *mixed* industries. In many studies of large metropolitan regions, the ultimate basis for determining the basic and service components of mixed industries takes on some form of *location quotient* (or concentration ratio). We now turn to a discussion of it.

Frequently, a study of a region's export-import relations begins at least on a *preliminary basis* with a simple analysis employing the location quotient. It is a simple device for comparing a region's percentage share of a particular activity with its percentage share of some basic aggregate. Specifically, it can be illustrated using the following formulation.

$$L_{ij}=(X_{ij}/X_{oj})/(X_{io}/X_{oo})$$

where the subscripts i, j refer to industry and region, respectively. The subscript o denotes summation; in the right-hand position it is summation over regions, in the left-hand position it is summation over industries (i.e., total employment or total earnings, depending on the definition of X). According to its numerical values, ratios greater than unity, for example, are taken to indicate an export or basic industry of a region. Therefore, the advantages of the location quotient method are its *simplicity* and the fact that it can be based on readily available data. Because of its simplicity, the location quotient is useful, especially, in the early exploratory stages of research.

However, the use of the location quotient to identify basic-service components of a region must be seriously qualified, since it makes implicit assumptions.[25] It assumes that, with reference to the mixed

25) See W. Isard, op. cit., pp. 123-126 and p. 195.

industry, local patterns of use and habits of consumptions are the same as average national ones, and that all local demands are served by local production. Clearly there are many instances in which either or both of these assumptions are erroneous.

In sum, the location quotient does not, of itself, tell us much. When the limitations of the available date are adequately recognized and conclusions are properly qualified, the location quotient can be of value. However, the key to the most fruitful use of it lies in *integration with other types* of regional analysis, above all things, when built into input-output studies.[26)]

VI

In this paper, we have discussed a concept of *multiplier* which is derived from the input-output model.[27)] As is crystal-clear, "sound regional analysis is interdependence analysis."[28)] The economic base model, which we discussed in the previous section, has turned out to be subject to major deficiencies. It fails to detect the interdependencies of the many sectors of the economy, and it is unable to identify impacts on each of the sectors individually. The approach which underscores these interdependencies, and incorporates them into a regional analytical too

26) D. H. Garnick has explored the conditions that would permit the cruder methods of correlation to substitute for the less data-parsimonious methods of input-output for purposes of multiregional projection. Although a *disaggregated* basic-service model, which he proposed, is able to discriminate the regional share effect for each of the residentiary industries, the problem still remains in how to classify all economic activities of a region into basic or service categories. See D. H. Garnick, "Disaggregated Basic-Service Models and Regional Input-Output Models in Multiregional Projections," Journal of Regional Science, vol. 9, (1969).

27) For the comments raised in this section, this writer is indebted to B. M. Renaud, who was his discussant at the First Pacific Regional Science Conference, which was held in Hawaii in 1969.

28) W. Isard, ob. cit., p. 743.

is the *input-output technique*. The immediate advantage of a regional input-output model over a basic-service model lies in the ability of the former to trace differential within-region impacts for exogenous changes in any of the final demands.

It should be noted, however, that even an input-output model has some limitations. When we consider the validity of empirical estimates of the coefficients, we are very familiar with the shortcomings of input-output analysis. Since other quantitative tools are now available for regional planning as well as for forecasting, we might ask ourselves again whether the results will justify another similar effort.[29] For all input-output models, the question centers around the use of input-output coefficients as meaningful and stable structural parameters. To summarize the difficulties: we assume linearity in production when economies of scale might be present; externalities due to the location of additional plants of the same industry or the agglomeration of different industries in the same location can hardly enter the analysis;[30] relative price changes do take place, even within short periods of time; it is difficult to take into account technological changes and their impact on the production coefficients; factor substitution is impossible.

An additional issue with interregional models is the stability of trading coefficients. Research costs often require that each local sector follows the average import pattern of the region as a whole. In practice, depending on the localization of an establishment in the center of the

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29) According to the Garnick's empirical studies, for example, location quotients for residentiary industries tend to be considerable clustering around unity and somewhat decreasing standard deviations over time. These results are not discouraging to the user of location quotients, particularly if he is willing to adjust for basic components and special regional patterns. See the table 2, in D. H. Garnick, op. cit., p.92.

30) The first attempt in this field was made by Isard and Kuenne, which we discussed in section IV.

region or on the borderline, we should observe very different trade patterns within the sector. We need to assume regional cost structures and transportation rates as given. But what happens with the opening of a new freeway right after completion of the study? An even more serious issue is the assumption that industries will not easily shift their sources of supply. But some economists have submitted empirical evidence showing that some firms tend to vary their regional sales and purchases with their level of output. It is important to investigate the empirical importance of these problems before embarking on a planning study.

Nonetheless, the general consensus, at least for national tables, is that an assumption of stability of the coefficients is valid for many *short-run* problems. Once the tables have been built, it is very interesting to manipulate them and to investigate the economic possibilities and problems ahead of the current period. Such manipulations can be grouped under three headings:

1) forecasting problems,
2) use of the model for planning,
3) use for the evaluation of stabilization policies.

For forecasting, input-output models have a strong appeal due to their ability to yield consistent forecasts and the fact that their *multipliers* give first static approximations to the impact of a change in the final demand vector. The problem is, then, to know how good these forecasts are.[31)]

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31) The best empirical evidence available is the discussion of the Dutch experience at the national level presented by Tilanus. He points out that when exogenous forecasts of the final demand vector are not good, forecasts of the intermediate demand vector are even worse. In the case of regional forecasts, there are additional chances for miscalculations since we are losing an opportunity that errors in disaggregated predictions will compensate each other in the grand total for the national economy. His time-series analysis of the input coefficients based on 13 annual tables shows that the dispersion of the observations around the trend is substantial. This disheartening

For planning purposes, an interregional input-output study appears quite useful for the evaluation of important developmental programs.[32) These programs give rise to significant interaction with the current structure of the economy and the patterns of trade. One must immediately admit, however, that large investment projects have an effect over long periods of time. A dynamic form of the interregional model appears necessary and a capital coefficient matrix would have to be built to take into account the general capital formation process, anticipated technological changes, population growth, demand behavior, present and future trade opportunities, interregional resource movements, and expected changes in the industrial composition of output.[33)]

It seems to the writer that, at the present time, the best use of an interregional model is as a laboratory for *research purposes.* This rather conservative attitude is based on what this writer could learn of the analytical—as against descriptive—use made of input-output studies. The lack of enthusiasm of empirical users may be explained by the high administrative costs of keeping an up-to-date table and more important that a general-purpose table does not exist.[34)] Once we have accepted

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fact implies that the tables have to be adjusted frequently. Mechanical adjustments like Stone's RAS method could possibly be used, but they have proved to be less effective than the empirical recomputation of the most strategic coefficients.

32) For example, T. A. Reiner discussed the spatial allocation criteria in terms of a hypothetical interregional model. The problem raised by Reiner seems to be of considerable use to the analysts, particularly the regional policy makers, when the valid interregional table is available. See T. A. Reiner, "Spatial Criteria for Programs to offset Military Cutbacks," Peace Research Society (International) Papers, vol. 3, (1965).

33) A recent empirical study of the California water programs along these lines has not been very conclusive. See W. Yep, "Economic Base and Projections of the California Framework Study: 1965—2020."

34) As for an empirical California-Washington interregional input-output model and its associated tests on the stability of the trade coefficients, see R.

$$r=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}}.$$

Since the sampling distribution of r for random samples from bivariate normal population is rather complicated, it is common practice to base tests concerning ρ on the statistic $\frac{1}{2}\cdot ln\frac{1+r}{1-r}$, whose distribution is approximately normal with the mean $\frac{1}{2}\cdot ln\frac{1+\rho}{1-\rho}$ and the variance $\frac{1}{n-3}$. Thus,

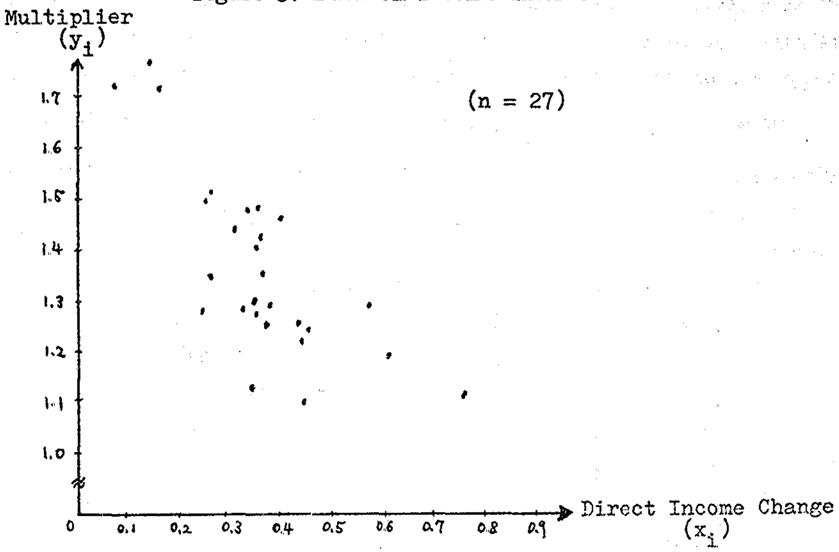
$$z=\frac{\frac{1}{2}\cdot ln\frac{1+r}{1-r}-\frac{1}{2}\cdot ln\frac{1+\rho}{1-\rho}}{1/\sqrt{n-3}}=\frac{\sqrt{n-3}}{2}ln\frac{(1+r)(1-\rho)}{(1-r)(1+\rho)}$$

can be looked upon as a value of a random variable having approximately the standard normal distribution. Using this approximation, we can test hypotheses concerning ρ, or calculate confidence intervals for ρ as will be shown below.

Suppose we want to determine on the basis of the data calculated by Hirsch whether there is an inverse relationship between the direct income change (i.e., the household row of the input coefficient table) and the income multiplier (i.e., the direct and indirect changes in income resulting from an increase of one dollar in the output of all the industries in the processing sectors).

First the calculation of r between two series (i.e., column 1 and column 4 in Table 4) yields $r=-0.743$, which indicates a negative association between the direct income change and the income multiplier. This association is also apparent from the scattergram of Figure 3.

Figure 3. Data on Income Interactions



Next substituting $r=-0.743$, n=27, and $\rho=0$ into the above equation for z, we get

$$z=\frac{\sqrt{24}}{2}\cdot ln\frac{0.257}{1.743}\doteqdot -4.653,$$

and since this is less than $z_{.025}=-1.96$, the null hypothesis of no correlation must be *rejected*. Hence we can conclude that there *is* an inverse relationship (or a negative association) between the direct income change and the income multiplier.[35]

The statistical testing in correlation analysis can be applied to any paired data given in Table 4. While the correlation coefficient between the income multiplier and the direct & indirect income change (i.e., column 2) is −0.620, which is also significant at an α of 0.05, the correlation coefficient between the income multiplier and the indirect income change alone (i.e., colmn 3) is 0.259, which is *not significant* at

35) It is highly significant even at an α of 0.01.

the same level of significance. These empirical findings lend additional statistical refinement to the original hypothesis that the direct income change is associated with the income multiplier having a downward slope.

When we let $t=\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$, then a value of t has the Student t distribution with n−2 degrees of freedom. Thus, using it, we can also test the null hypothesis: $\rho=0$. In bringing this paper to a close, we summarize our numerical results as follows:

Col.1 & col.4; $r=-0.743$ $t=-5.554$ (rejected),
Col.2 & col.4; $r=-0.620$ $t=-3.954$ (rejected),
Col.3 & col.4; $r=+0.259$ $t=+1.340$ (accepted),
Col.5 & col.8; $r=-0.593$ $t=-3.684$ (rejected),
Col.6 & col.8; $r=-0.560$ $t=-3.379$ (rejected),
Col.7 & col.8; $r=-0.338$ $t=-1.795$ (accepted),

It should be fully appreciated that these tests are approximate and we had to assume that the given data can be looked upon as a *random sample* from a bivariate normal population.[36]

36) Incidentally, Hirsch has also pointed out that St. Louis interindustry multipliers of model 1 (i.e., col.4) are highly correlated with those of model 2 (i.e., col.8). In this context, he states that the following regression equation well describes the 1955 relationship:

$y=0.35+1.29x$,

where y is the 1955 St. Louis interindustry income multiplier of model 2, and x is that of model 1. Yet, the computations for OLS by this writer from materials in Table 4 have brought about the following somewhat different results:

$y=1.45+0.18x$,
(1.88)(20.85)

where $r=0.97$ and $r^2=0.95$.