LETTER

Coherence Resonance in Propagating Spikes in the FitzHugh-Nagumo Model

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SUMMARY Coherence resonance in propagating spikes generated by noise in spatially distributed excitable media is studied with computer simulation and circuit experiment on the FitzHugh-Nagumo model. White noise is added to the one end of the media to generate spikes, which propagate to the other end. The mean and standard deviation of the interspike intervals of the spikes after propagation take minimum values at the intermediate strength of the added noise. This shows stronger coherence than obtained in the previous studies.

key words: *stochastic resonance, coherence resonance, excitable media, FitzHugh-Nagumo model, spike propagation*

Stochastic resonance in excitable media is of wide interest since it may be related to sensory signal processing in nervous systems [1]. Recently, it was shown that coherence resonance (stochastic resonance without input signals), which was found in some limit cycle models [2], occurs also in excitable media [3]–[12]. That is, the coherence or regularity of the spikes generated by additive noise in excitable media is optimal at intermediate noise strength.

In this letter coherence resonance in spatially distributed excitable media is studied with computer simulation and circuit experiment on the FitzHugh-Nagumo model, a simple model of a nerve fiber [13], [14]. White noise is added to the one end of the media, by which spikes are generated and propagate to the other end. It is shown that the mean and standard deviation of the interspike intervals of the spikes after propagation take minimum values at intermediate noise strength. This is stronger coherence than that obtained in the previous studies on excitable media, in which some relative measures of coherence, e.g., the product of the height of the peak and the quality factor of the power spectrum, the coefficient of variation and the correlation time were used. Although similar non-monotonic relations of the firing frequency to the numbers of ion channels have been obtained in computer simulation on stochastic versions of the Hodgkin-Huxley model, in which fluctuations in ion channel dynamics are taken into account [15], [16], the mechanism causing this coherence resonance is different from them.

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First, the results of computer simulation on the one-dimensional FitzHugh-Nagumo model are shown.

$$dv/dt = \partial^2 v/\partial x^2 - v(v-a)(v-1) - w + n\delta(x)$$

$$dw/dt = \varepsilon(v - \gamma w) \quad (a = 0.2, \varepsilon = 0.003, \gamma = 0.5, 0 \le x \le 30) \qquad (1)$$

where Gaussian white noise n with zero mean and strength σ ($E\{n(t)n(t')\} = \sigma^2\delta(t-t')$) is added to v at the one end ($x = 0$). Spikes are generated by the noise and propagate to the other end. Equation (1) is numerically calculated by the simple Euler method with $\Delta x = 1.0$ and $\Delta t = 0.2$. The time series of v at the noise-added point ($x = 0$) and after propagation ($x = 25$) with $\sigma = 0.2$, 0.4 and 0.6 are shown in Fig. 1. The spikes of proper shape are obtained after propagation ($x = 25$) owing to the wave shaping action of excitable media [17], while there are large variations due to the noise at $x = 0$. Figure 2 plots the mean and standard deviation (S.D.) of 10000 interspike intervals of the propagating spikes at $x = 25$ against the noise strength σ. Both take minimum values at $\sigma \cong 0.38$. This means the frequency and regularity of the spikes are absolutely highest at the intermediate noise strength. The coefficient of variation (the ratio of the standard deviation to the mean) of the interspike intervals is also minimum at the same noise strength $\sigma \cong 0.38$, though not shown here. Note that the values of the noise strength are smaller than the amplitude of the spikes and are considered to be physically relevant.

This coherence resonance can be obtained in wide parameter ranges. Figure 3 shows changes in the optimal noise strength σ_{opt} and the mean interspike interval T_{opt} by adding constant input I at $x = 0$ to the right-hand side of dv/dt in Eq. (1). The optimal noise strength and the mean interspike interval are small at $0.3 \le I \le 1.3$, where the spikes are periodically generated without noise in the single element.

Next, experiment on Nagumo's active transmission line [14], an analog circuit for the FitzHugh-Nagumo model is done. The N-shaped nonlinear current device (T.D.) and inductor are constructed with operational amplifiers [18], as shown in Fig. 4. The Nagumo's active line is made by coupling 20 elements with resistors. The value of V_m is set to be 4.9 V so that the elements are mono-stable. White noise source is added to V_m

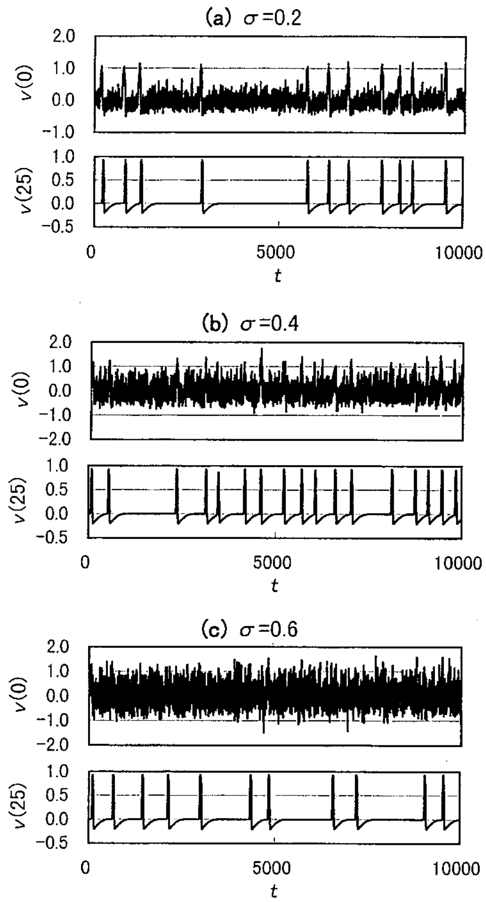

Fig. 1 Time series of v at the noise-added point ($x = 0$) and after propagation ($x = 25$) with $\sigma = 0.2$ (a), 0.4 (b) and 0.6 (c) (the FitzHugh-Nagumo model).

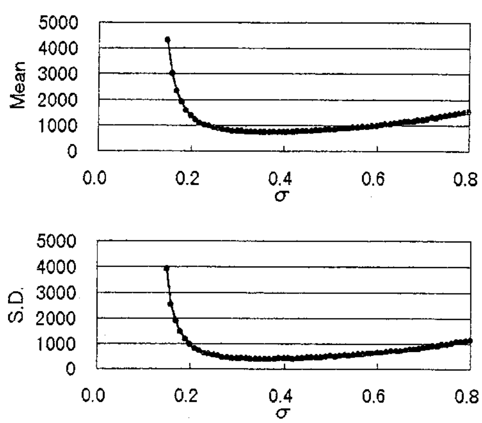

Fig. 2 Mean and standard deviation (S.D.) of 10000 interspike intervals of the propagating spikes at $x = 25$ vs. noise strength σ (the FitzHugh-Nagumo model).

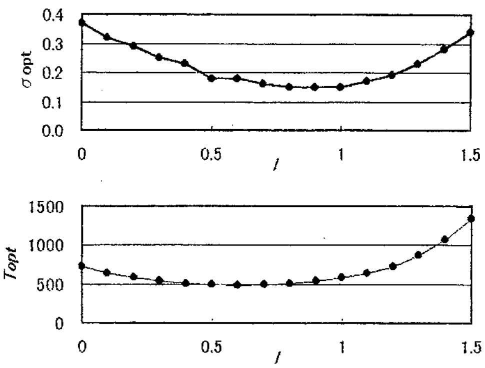

Fig. 3 Optimal noise strength σ_{opt} and mean interspike interval T_{opt} vs. constant input I (the FitzHugh-Nagumo model).

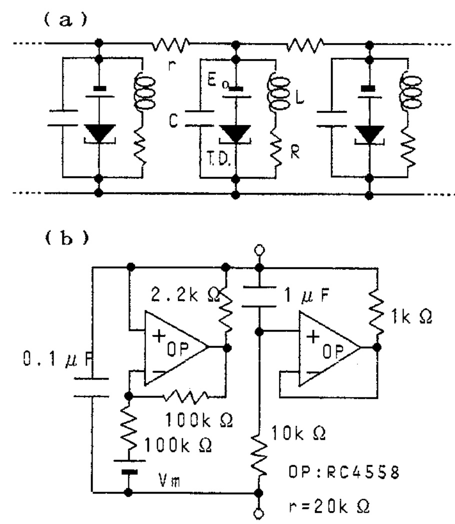

Fig. 4 Nagumo's active transmission line (a) and an analog circuit for one element with OP amps (b).

in the first element, by which spikes are generated and propagate in the line. Figure 5 shows the time series of the voltage in the 1st and 20th element with the noise strength $\sigma = 0.5$, 2.0 and 3.5 V. The mean and S.D. of the interspike intervals in the 20th element recorded during 100 sec are plotted against the noise strength σ in Fig. 6. The mean and S.D. of the interspike intervals after propagation take minimum values at the intermediate values $\sigma \cong 2.0$–2.5 V of the noise strength.

It was shown that the mean and standard deviation of the interspike intervals of the propagating spikes generated by point stimulus in excitable media take minimum values at the intermediate levels of noise strength. The previous studies on coherent resonance in the single element of excitable media have shown that the mean interspike interval decreases monotonically as the noise strength increases [4], [8], [9]. The results obtained in this study show the existence of stronger coherence than those in these studies, and are also different from the previous results on coherence resonance and noise-sustained patterns in spatially distributed excitable media [7], [10], [11], [19].

The mechanism of this coherence resonance is a combination of the decrease in the interspike intervals of the spikes due to the noise, which is attributed to the mean first passage time for the Ornstein-Uhlenbeck

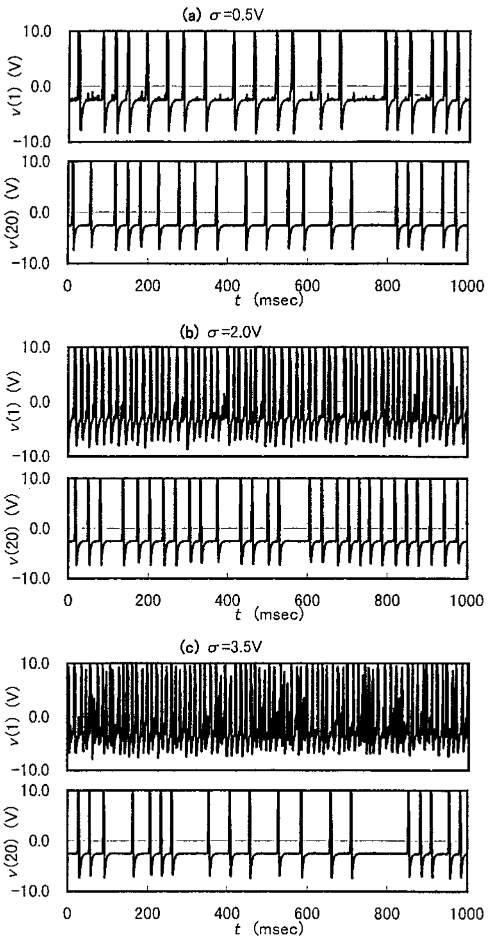

Fig. 5 Time series of the voltage in the noise-added (1st) element and 20th element with $\sigma = 0.5\,\mathrm{V}$ (a), 2.0 V (b) and 3.5 V (c) (Nagumo's active line).

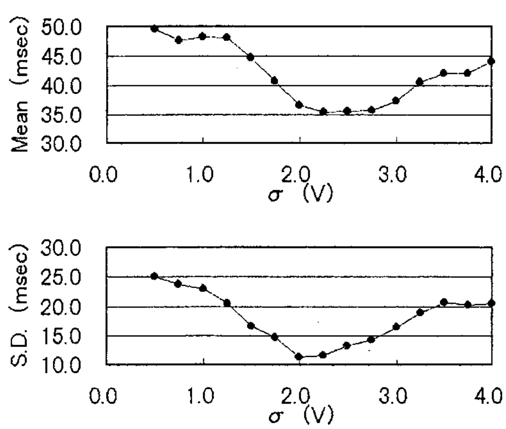

Fig. 6 Mean and S.D. of the interspike intervals of the propagating spikes in the 20th element vs. noise strength σ (Nagumo's active line).

process and a double-well potential system [20], and the failure of spike propagation due to the refractory period [17]. That is, as the noise strength increases, the interspike intervals at the stimulus point decrease, but the spikes with the interval to the previous one smaller than the refractory period fail to propagate. This failure of the spike propagation makes the mean and S.D. of the interspike intervals larger. It is difficult to see the spike failure due to the refractory period in the single element model since the noise gives large variations at the stimulus point, as can be seen in Figs. 1 and 5. The wave shaping action according to spike propagation in spatially distributed excitable media makes this coherence resonance clear.

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