COMPARATIVE STATIC ANALYSIS OF THE FIRM

—— in conjunction with the Moses model ——

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I

One of the conclusions of Moses' 1958 paper on location theory of the firm was that: "... the optimum location is seen finally to depend on the following factors; base prices on inputs; transportation rates on inputs and on the final product; the geographic positions of materials and markets; the production function; the demand function."[1)]

Although his analytical method has not progressed beyond the partial equilibrium analysis, it should be noted that the main objective in his paper was to make the theory of location an integral part of the theory of production and to investigate the implications of factor substitution for the locational equilibrium of the firm.

The purpose of the present paper is to clarify the implications of the equilibrium point in the Moses model. The problem will be approached by the *comparative statics*, which enables us to investigate the qualitative direction of movement of the equilibrium point. Section II begins with

1) Moses (4, p, 269),

the mathematical reformulation of the Moses model. Section III will be devoted to the methods of comparative static analysis. Finally, in section IV, some theoretical conclusions will be clarified in the light of our framework.

II

In order to deal with the comparative-static analysis in a spatial setting, let us reformulate the Moses model as follows. As a common assumption, we postulate the simple case of a firm which employs two transportable inputs to produce a single product that is sold in a single market point. Figure 1 depicts the locational problem where M_1 and M_2 are the sites of the two materials and C is the market point.

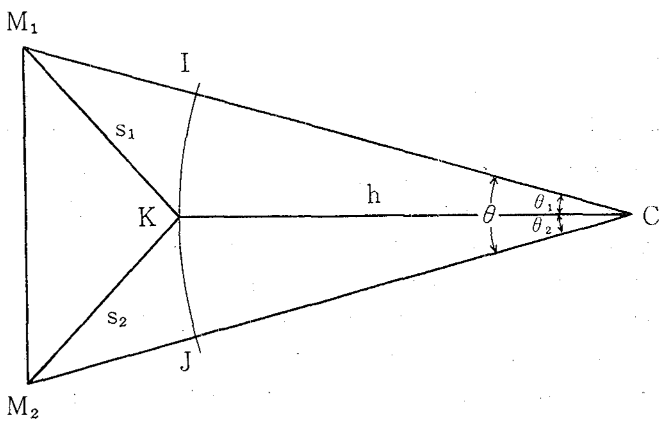

Figure 1

Since the distances M_1M_2, M_1C and M_2C are known, the given sites of the two materials and the market point can be expressed in terms of the two-dimensional coordinate system. Without loss of generality, we may assign the coordinates $(0,0)$, $(x_1,0)$ and (x_2,y_2) to the points of C, M_1 and M_2, respectively, which is shown in Figure 2,

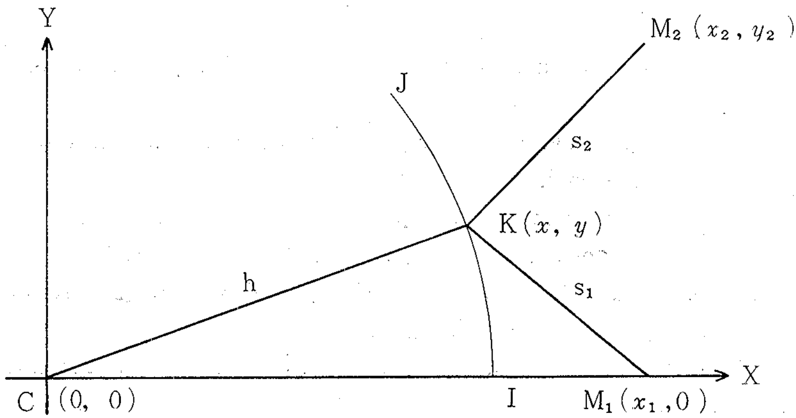

Figure 2

Let K represent the firm's location point, whose coordinates (x,y) are unknown. According to Moses, the distance that the final product must be shipped is assumed to be constant. Thus it can be expressed as;

$$\overline{KC}=h, \text{ or } x^2+y^2=h^2.$$

Geometrically, it corresponds to an arc, which is a segment of the circle with center at C and radius h, as shown in Figure 1 and 2.

Now let us specify the locational situation of our firm. Under the assumptions given above, our firm is going to find its best location at some point on the arc $(\widehat{IJ})$ which links the site of the first raw material, M_1, and that of the second raw material, M_2. The following notation will be used:

(parameters)

p_1 = the price of the first raw material at its source,

p_2 = the price of the second raw material at its source,

r_1 = the transport rate on the first input,

r_2 = the transport rate on the second input.

(unknowns)

s_1 = the distance from M_1 to the locus of production of the final product,

s_2 = the distance from M_2 to the locus of production of the final product,

v_1 = the amount of the input of the first material,

v_2 = the amount of the input of the second material.

It should be noted that, in those symbols, p_i and r_i $(i=1,2)$ are to be considered as parameters for the given firm.

Furthermore, we postulate that the firm's production function is continuously twice differentiable and includes only two input-variables. Let this function be expressed as

(1) $q=f(v_1,v_2)$

where q=the amount of the final product.

With transportation cost on the final product fixed, we can express the cost function of the firm associated with a fixed level of output $q=\bar{q}$ as;

(2) $c=(\bar{p}_1+\bar{r}_1s_1)v_1+(\bar{p}_2+\bar{r}_2s_2)v_2.$

Therefore, our problem for determining the optimum location of the firm can well be formulated as follows:

(3) Minimize $c=(\bar{p}_1+\bar{r}_1s_1)v_1+(\bar{p}_2+\bar{r}_2s_2)v_2$

(4) subject to $\bar{q}=f(v_1,v_2)$

where

(5) $s_1=\sqrt{(x-x_1)^2+y^2}$,

(6) $s_2=\sqrt{(x-x_2)^2+(y-y_2)^2}$.

In order to deal with the one equality-constrained minimization problem, let us form the Lagrangean function, L, by subtracting the constraint (4), multiplied by an unknown new variable, λ, from the cost function (3).

(7) $L=(\bar{p}_1+\bar{r}_1 s_1)v_1+(\bar{p}_2+\bar{r}_2 s_2)v_2-\lambda\{f(v_1,v_2)-\bar{q}\}$

However, by virtue of the relationship of $x^2+y^2=h^2$, the unknown variables s_1 and s_2 cannot be dealt with as independent ones in equation (7). Regarding the variation in x (i.e., the abscissa of the point K in Figure 2) as the location variable,[2] we can define the moving point K on the arc $\overset{\frown}{\mathrm{IJ}}$.

Substituting $x^2+y^2=h^2$ into (7) yields

(8) $$L=(\bar{p}_1+\bar{r}_1\sqrt{\bar{x}_1^2+\bar{h}^2-2\bar{x}_1 x})v_1$$
$$+(\bar{p}_2+\bar{r}_2\sqrt{\bar{x}_2^2+\bar{y}_2^2+\bar{h}^2-2\bar{x}_2 x-2\bar{y}_2\sqrt{\bar{h}^2-x^2}})v_2-\lambda\{f(v_1,v_2)-\bar{q}\}.$$

Since the equation (8) can be viewed as the function of unknown v_1, v_2, x, λ, under the given values of parameters $\bar{p}_1, \bar{p}_2$, $\bar{r}_1, \bar{r}_2$ and exogenously given values of $\bar{x}_1, \bar{x}_2$ and $\bar{y}_2$.

In order to make a clear distinction between the unknowns and the parameters, let us rewrite the Lagrangean form as:

(9) $L=g(v_1, v_2, x, \lambda; p_1, p_2, r_1, r_2)$.

If we adopt the following notation,

2) The location variable (i.e., x in our framework) corresponds to the angle θ_1 in the Moses terminology. See Figure 1.

$z_1=v_1,\ z_2=v_2,\ z_3=x,\ z_4=\lambda,$

$\alpha_1=p_1,\ \alpha_2=p_2,\ \alpha_3=r_1,\ \alpha_4=r_2,$

then our Lagrangean form can also be written as:

$$(10)\qquad L=g(z_1,\ z_2,\ z_3,\ z_4;\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4).$$

III

Comparative statics, as the name suggests, is concerned with the comparison of different equilibrium points that are associated with different sets of values of parameters. For purposes of such a comparison, we always start by assuming a given initial equilibrium point.[3)]

Let us review the methods of comparative-static analysis briefly in this section. In our model, such an initial equilibrium will be represented by determinate values of unknown variables $(z_1,\ z_2,\ z_3,\ z_4)$ for preassigned values of parameters $(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$.

Let $L=g(z_1,\ z_2,\ z_3,\ z_4;\ \alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4)$ be a defined function with continuous second order partial derivatives of all kinds in an open region S. Then a point (Z^0) in S affords a relative minimum to L provided that

$$(11)\qquad g\ (z_1,z_2,z_3,z_4;\alpha_1,\alpha_2,\alpha_3,\alpha_4)\geqq g\ (z_1^0,z_2^0,z_3^0,z_4^0;\alpha_1,\alpha_2,\alpha_3,\alpha_4)$$

for sufficiently close values of (Z).

It is *necessary* in order for this to be true that

$$(12)\qquad \frac{\partial L}{\partial z_i}=g^0_{z_i}=0\qquad (i=1,2,3,4),$$

and

$$(13)\qquad \sum_{i=1}^{4}\sum_{j=1}^{4}g^0_{z_iz_j}h_ih_j>0,\ \text{for not all } h\text{'s equal to zero.}$$

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3) Chiang (1, pp. 132-133)

where (h_1, h_2, h_3, h_4) are arbitrary numbers, and the partial derivatives are evaluated at the point (Z^0).[4] In other words, the appropriate quadratic form must be *positive definite*.[5]

Hence, an initial equilibrium values (i.e., z_i^0; $i=1,2,3,4$) can be derived from the first order conditions (12), which involve some or all of unknowns (z_i) and the parameters (α_k).

Now if we let a disequilibrating change occur in the model--- in the form of a variation in the value of some parameter (α_k)---, the initial equilibrium will, of course, be upset. As a result, the various unknowns (z_i) must undergo certain adjustments. If it is assumed that a new equilibrium point relevant to the new values of the data can be defined and attained, the question posed in the comparative-static analysis is; how would the new equilibrium compare with the old?

The problem under consideration is essentially one of finding a *rate of change* (say, $\frac{\partial z_i^0}{\partial \alpha_k}$); the rate of change of the equilibrium value of an endogenous variable (z_i^0) with respect to the change in a particular parameter (α_k). For this purpose, differentiating (12) totally with respect to α_k, we can derive the following relations:

$$(14) \qquad \sum_{j=1}^{4} g^0_{z_i z_j} \frac{\partial z_j}{\partial \alpha_k} = -g^0_{z_i \alpha_k} \qquad (i=1,2,3,4).$$

The solution for this simultaneous equation system can be expressed as:

4) See Samuelson (6, pp. 359-361).

5) In this case,

$$Q(h_1, h_2, h_3, h_4) = [h_1\ h_2\ h_3\ h_4] \begin{pmatrix} g_{z_1z_1} & g_{z_1z_2} & g_{z_1z_3} & g_{z_1z_4} \\ g_{z_2z_1} & g_{z_2z_2} & g_{z_2z_3} & g_{z_2z_4} \\ g_{z_3z_1} & g_{z_3z_2} & g_{z_3z_3} & g_{z_3z_4} \\ g_{z_4z_1} & g_{z_4z_2} & g_{z_4z_3} & g_{z_4z_4} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{pmatrix} > 0.$$

(15)
$$\frac{\partial z_j}{\partial \alpha_k} = -\frac{\sum_{i=1}^{4} g_{z_i \alpha_k} \cdot \Delta_{ij}}{\Delta},$$

where

$$\Delta = \begin{vmatrix} g_{z_1 z_1} & g_{z_1 z_2} & g_{z_1 z_3} & g_{z_1 z_4} \\ g_{z_2 z_1} & g_{z_2 z_2} & g_{z_2 z_3} & g_{z_2 z_4} \\ g_{z_3 z_1} & g_{z_3 z_2} & g_{z_3 z_3} & g_{z_3 z_4} \\ g_{z_4 z_1} & g_{z_4 z_2} & g_{z_4 z_3} & g_{z_4 z_4} \end{vmatrix},$$

and Δ_{ij} is the cofactor of the element in the i-th row and j-th column of Δ.

Obviously, in equation (15), each unknown derivative depends upon an (4×5) infinity of possible values. If the various determinants were expanded out, a sum of 4! terms would appear in the denominator and in the numerator. Therefore, unless some *a priori* restrictions are placed upon the nature of the elements involved in these determinants, no useful results can be derived.

Fortunately, however, if we take into account the relationship between (13) and (14), we can evaluate ---without proceeding in a fairly straightforward manner--- these complicated expressions to some extent. Namely, multiplying the i-th equation in (14) by $\frac{\partial z_i}{\partial \alpha_k}$, we have:

(16)
$$\frac{\partial z_i}{\partial \alpha_k} \sum_{j=1}^{4} g^0_{z_i z_j} \frac{\partial z_j}{\partial \alpha_k} = -g^0_{z_i \alpha_k} \frac{\partial z_i}{\partial \alpha_k}.$$

Then, adding them up over $i (i=1,2,3,4)$, we have:

(17)
$$\sum_{i=1}^{4} \sum_{j=1}^{4} g^0_{z_i z_j} \frac{\partial z_i}{\partial \alpha_k} \frac{\partial z_j}{\partial \alpha_k} = -\sum_{i=1}^{4} g^0_{z_i \alpha_k} \frac{\partial z_i}{\partial \alpha_k}.$$

As stated earlier, the optimum value of z_i^0 $(i=1, 2, 3, 4)$ must always satisfy the inequality (13) for any arbitrary given values of h's. Therefore, if we specifically regard $\frac{\partial z_i}{\partial \alpha_k}$ as h_i for any i, i.e.,

$$(18) \qquad h_i = \frac{\partial z_i}{\partial \alpha_k} \qquad (i=1,2,3,4)$$

then it must hold that

$$(19) \qquad \sum_{i=1}^{4} \sum_{j=1}^{4} g^0_{z_i z_j} \frac{\partial z_i}{\partial \alpha_k} \frac{\partial z_j}{\partial \alpha_k} > 0.$$

This means, from (17), that the following inequality

$$(20) \qquad \sum_{i=1}^{4} g^0_{z_i \alpha_k} \frac{\partial z_i}{\partial \alpha_k} < 0$$

must always hold for not all $\frac{\partial z_i}{\partial \alpha_k}$ equal to zero.

IV

Let us go back to the Moses model again and examine the nature of its solution by the comparative-static analysis. Recall that our cost-minimizing function was formulated as (8) in section II. Conditions (12) applied to our problem are:

$$\begin{cases} \dfrac{\partial L}{\partial v_1} = L_1 = (p_1 + r_1\sqrt{x_1^2 + h^2 - 2x_1 x}) - \lambda f_{v1} = 0, \\[2ex] \dfrac{\partial L}{\partial v_2} = L_2 = (p_2 + r_2\sqrt{x_2^2 + y_2^2 + h^2 - 2x_2 x - 2y_2\sqrt{h^2 - x^2}}) - \lambda f_{v2} = 0, \\[2ex] \dfrac{\partial L}{\partial x} = L_x = -\dfrac{r_1 v_1 x_1}{\sqrt{x_1^2 + h^2 - 2x_1 x}} + \dfrac{r_2 v_2 \left(-x_2 + \dfrac{y_2 x}{\sqrt{h^2 - x^2}}\right)}{\sqrt{x_2^2 + y_2^2 + h^2 - 2x_2 x - 2y_2\sqrt{h^2 - x^2}}} = 0, \\[2ex] \dfrac{\partial L}{\partial \lambda} = L_\lambda = -f(v_1, v_2) + \bar{q} = 0. \end{cases}$$

With an aid of inequality (20), we can draw at least the following theoretical conclusions:

Case 1

Carrying out the appropriate calculations, we can show that

$$L_{v_1p_1}=1,\ L_{v_2p_1}=0,\ L_{xp_1}=0,\ L_{\lambda p_1}=0.$$

Then, from (20),

$$(21)\qquad \frac{\partial v_1}{\partial p_1}<0,$$

which means that as the price of the first raw material goes up, the amount of the input of its material always decreases.

Case 2

Similarly, with the derived relations of

$$L_{v_1p_2}=0,\ L_{v_2p_2}=1,\ L_{xp_2}=0,\ L_{\lambda p_2}=0,$$

it can be readily verified that

$$(22)\qquad \frac{\partial v_2}{\partial p_2}<0,$$

for the change of p_2.

Case 3

By the same token,

$$L_{v_1r_1}=\sqrt{x_1^2+h^2-2x_1x}\ ,\quad L_{v_2r_1}=0\,,$$

$$L_{xr_1}=-\frac{v_1x_1}{\sqrt{x_1^2+h^2-2x_1x}}\ ,\quad L_{\lambda r_1}=0,$$

The necessary condition for minimizing cost (20) becomes in this case,

$$\sqrt{x_1^2+h^2-2x_1x}\ \frac{\partial v_1}{\partial r_1}-\frac{v_1x_1}{\sqrt{x_1^2+h^2-2x_1x}}\ \frac{\partial x}{\partial r_1}<0.$$

By rearranging it we have

$$(23)\qquad \frac{\partial v_1}{\partial r_1}<\frac{v_1x_1}{x_1^2+h^2-2x_1x}\cdot\frac{\partial x}{\partial r_1},$$

or,

$$(23)'\qquad \frac{\partial v_1}{\partial r_1}<\frac{v_1x_1}{(x_1-x)^2+y^2}\cdot\frac{\partial x}{\partial r_1}.$$

Although (23)′ shows the necessary condition for the cost minimization problem, we cannot state unambiguously the direction of changes of v_1 and x from this result alone.

Case 4

Similarly, with the derived relations of

$$L_{v_1r_2}=0,\quad L_{v_2r_2}=\sqrt{x_2^2+y_2^2+h^2-2x_2x-2y_2\sqrt{h^2-x^2}},$$

$$L_{xr_2}=\frac{v_2\left(-x_2+\frac{y_2x}{\sqrt{h^2-x^2}}\right)}{\sqrt{x_2^2+y_2^2+h^2-2x_2x-2y_2\sqrt{h^2-x^2}}},\qquad L_{\lambda r_2}=0,$$

we have

$$(24)\qquad \frac{\partial v_2}{\partial r_2}<\frac{v_2\left(x_2-\frac{y_2x}{\sqrt{h^2-x^2}}\right)}{x_2^2+y_2^2+h^2-2x_2x-2y_2\sqrt{h^2-x^2}}\cdot\frac{\partial x}{\partial r_2},$$

or,

$$(24)'\qquad \frac{\partial v_2}{\partial r_2}<\frac{v_2\left(x_2-\frac{y_2x}{y}\right)}{(x-x_2)^2+(y-y_2)^2}\cdot\frac{\partial x}{\partial r_2}.$$

It means that whether or not v_2 will increase when r_2 changes depends upon not only the algebraic difference between x_2 and $\frac{y_2 x}{y}$, but also the $\frac{\partial x}{\partial r_2}$ at the equilibrium point.[6]

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4. Moses, Leon N., "Location and the Theory of Production," Quarterly Journal of Economics, Vol. 72, 1958. (Reprinted in Dean, et. al.)
5. Sakashita, N., "Production, Demand Function and Location Theory of the Firm," P. E. C. R. S. A. Vol. 20, 1968.
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6) While the sign of the denominator on the right hand side of (24)′ is always positive, the numerator changes the sign according to whether $x_2 - \frac{y_2 x}{y} \gtreqless 0$. However, since $x_2 - \frac{y_2 x}{y} > 0$ is equivalent to $\frac{y_2}{x_2} < \frac{y}{x}$ in this context, it means that the firm's location point $K(x, y)$ is inside the triangle CM_1M_2 in Figure 2.

CORRIGENDA

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This is a list of the errors and corrections for my paper entitled ON THE SYSTEM OF INTERREGIONAL COMMODITY FLOWS published in the Annual Report, Vol. 12, by the Department of Economics, Kagawa University in 1972.

Page & Line Number	Wrong	Correct
p. 75 l. 17	model-	model.
p. 76 l. 2	maixmizing	maximizing
p. 77 fig.1	Prefernce	Preference
p. 77 fig.1	Cost.	Coef.
p. 78 l. 17	follows:	follows: see Fig 2.
p. 79 fig.2	Propencity	Propensity
p. 81 l. 11	$x^{n}_{i\cdot}$ $x^{m}_{\cdot j}$	$x^{m}_{i\cdot}$ $x^{m}_{\cdot j}$
p. 81 l. 25	Poleenske	Polenske
p. 82 l. 21	point-to point	point-to-point
p. 84 l. 12	$(x^{m}_{\cdot r}\ q^{m}_{\cdot r})$	$(x^{m}_{\cdot r}\ q^{m}_{ir})$
p. 85 l. 3	exogenousvariables	exogenous variables
p. 88 l. 11	x_j	X_j
p. 88 l. 12	$x_1,\ x_2, \cdots,\ x_h$	$X_1,\ X_2, \cdots\cdots, X_h$
p.88 l. 24	p_{jn}	p_{jn_j}
p. 88 l. 27	$\frac{n_j}{\sum_{j=1}^{n} n_j} = \frac{m}{h_m}$	$\frac{n_j}{\sum_{j=1}^{h} n_j} = \frac{m}{hm}$
p. 89 l. 16	$p_1,\ p_2\ \ p_3.$	$p_1,\ p_2,\ p_3.$
p. 89 l. 18	assign to	assign 1 to
p. 90 l. 18	X^w	X^m

Page & Line Number	Wrong	Correct
p. 90 l. 21	inversesquare	inverse square
p. 91 l. 8	x_i^m,	$x_{i\cdot}^m$,
p. 91 l. 18	thers	there
p. 91 l. 23	x_j^m.	$x_{\cdot j}^m$
p. 92 l. 9	Y_i^m	Y_j^m
p. 92 l. 19	from	form
p. 92 l. 22	how to	how the
p. 93 l. 14	singlecommodity	single commodity
p. 93 l. 19	$X^m C x_{11}^m$	$X^m C x_{11}^m$
p. 93 l. 20	$X^m - x_{11}^m C x_{12}^m$	$X^m - x_{11}^m C x_{12}^m$
p. 94 l. 13	$p^{ij}_{\;m}$	p_{ij}^m
p. 94 l. 16	x_{ji}^m	x_{ij}^m
p. 94 l. 18	p_{ij}	p_{ij}^m
p. 95 l. 17	$-\sum_i x_{ij}^m$	$-\sum_j x_{ij}^m$
p. 95 l. 17	$\sum_i \lambda_j^{(2)}(Y_i^m$	$\sum_j \lambda_j^{(2)}(Y_j^m$
p. 96 l. 1	$-u^m c_{ij}^m$	$-\mu^m c_{ij}^m$
p. 96 l. 6	$-u^m c_{ij}^m$	$-\mu^m c_{ij}^m$
p. 97 l. 2	(51)	(51),
p. 97 l. 24	$X_l^m - \sum_j x_{lj}^m$	$X_1^m - \sum_j x_{1j}^m$
p. 97 l. 24	x_{lq}^m	x_{1q}^m
p. 98 l. 1	$Y_l^m - \sum_i x_{il}^m$	$Y_1^m - \sum_i x_{i1}^m$
p. 98 l. 11	x_{lq-1}^m	$x_{1,q-1}^m$
p. 98 l. 12	$i=1,2,.,\ 2^q$	$i=1,2,\cdots,2q$

Page & Line Number	Wrong	Correct
p. 98 l. 17	$\begin{pmatrix} \partial h^2/\partial x^m_{qq} \\ \partial h^{2q}/\partial x^m_{qq} \end{pmatrix}$	$\begin{pmatrix} \partial h^2/\partial x^m_{qq} \\ \vdots \\ \partial h^{2q}/\partial x^m_{qq} \end{pmatrix}$
p. 98 l. 17	$\begin{pmatrix} h^2_{x^m_{11}}, \cdots\cdots, h^2_{x^m_{qq}} \\ h^{2q}_{x^m_{11}}, \cdots\cdots, h^{2q}_{x^m_{qq}} \end{pmatrix}$	$\begin{pmatrix} h^2_{x^m_{11}}, \cdots\cdots, h^2_{x^m_{qq}} \\ \vdots \quad \vdots \\ h^{2q}_{x^m_{11}}, \cdots\cdots, h^{2q}_{x^m_{qq}} \end{pmatrix}$
p. 99 l. 4	Lagramgean	Lagrangean
p. 99 l. 22	Namely	Namely,
p. 100 l. 9	$0 - C^m_{12}$	$0 - C^m_{13}$
p. 100 l. 19	( )'	( )
p. 100 l. 21	( )'	( )
p. 101 l. 3	$\sum_{ik} \sum_{jl}$	$\sum_{i \neq k} \sum_{j \neq l}$
p. 101 l. 9	$j = 1$	$j = l$
p. 101 l. 10	$K \neq 1$	$K \neq l$
p. 101 l. 16	$f(X + dX)$	$f(X^* + dX)$
p. 101 l. 18	u^m	μ^m
p. 103 l. 4	$\sum_j x^m_{ij}$	$\sum_j x^m_{ji}$
p. 103 l. 4	$\sum_i x^n_{ij}$	$\sum_j x^n_{ij}$
p. 103 l. 18	$-\sum_j x^m_{ij}$	$-\sum_j x^m_{ji}$
p. 104 l. 1	where a 1	where a 1
p. 104 l. 6	$\sum_j$ exp $(\sum_m \gamma^n_i$	$\sum_j$ exp $(\sum_m \gamma^m_i$
p. 104 l. 8	$\sum_i$ exp $(\cdots\cdots$	$\sum_i$ exp $(\cdots\cdots$
p. 104 l. 9	a^i_{mm} exp$(\cdots\cdots$	a^i_{mm} exp $(\cdots\cdots$
p. 104 l. 11	exp $(\sum_m$	exp $(\sum_n$
p. 104 l. 15	$\Pi_m(\varepsilon^m_{ij})$	$\Pi_m(\varepsilon^m_i)$

Page & Line Number	Wrong	Correct
p. 104 l. 19	$a^i_{mn} S^n_i \sum_j$	$a^i_{mn} \delta^n_i \sum_j$
p. 105 l. 14	eq. (72)	eq. (72)′
p. 107 l. 13	achive	achieve
p. 108 l. 29	first-and	first- and
p. 109 l. 7	a objective	an objective