KING SOLOMON'S DILEMMA: A SIMPLE SOLUTION

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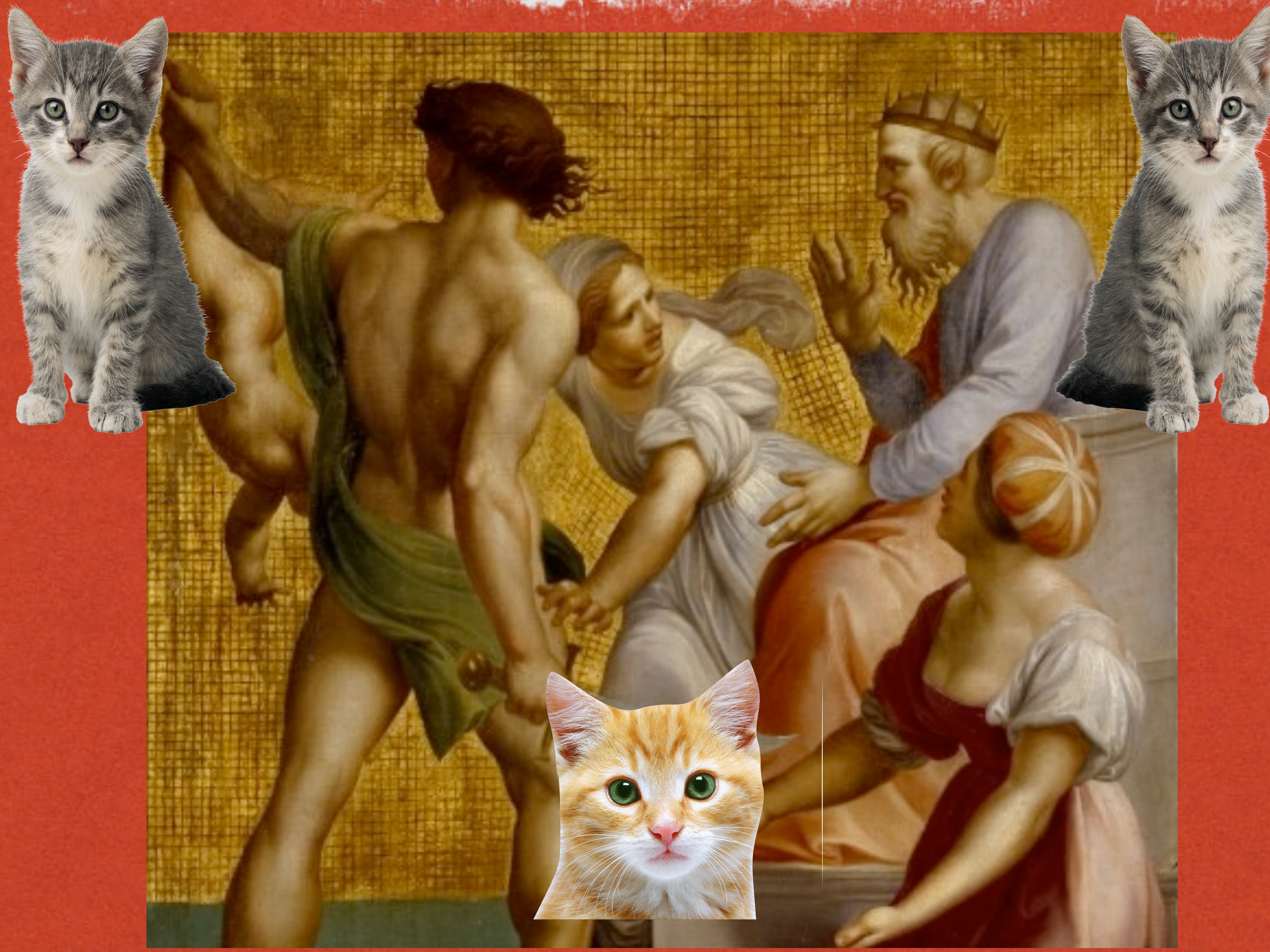
KING SOLOMON'S DILEMMA

- Problem
- Solution
- Comparison

PROBLEM



Giuseppe Cades: Judgement of Solomon; altered

THE JUDGMENT OF SOLOMON

MODEL

- n agents, 1,...., n
- k indivisible objects, where $k<n$
- At Stage 0, God announces (v, H),
  - $v = (v_1,...., v_n)$; v_i is agent i's valuation
  - $i \in H$ if v_i is among the top k valuations
- The problem is to allocate the k objects to the agents in H.

INFORMATIONAL ASSUMPTION

- The planner (King) does not observe (v, H).
- The planner and the agents know:

  if $i \in H$ and $j \notin H$, then $v_i - v_j > \delta > 0$.
- [Incomplete Info] Each agent i observes:
  - v_i, own valuation,
  - whether $i \in H$ or not.

SOLUTION

A FIRST ATTEMPT

- An auction?
  - For example, the (k+1)st-price auction, e.g., 2nd-price auction:
    - If i is among the k highest bidders, i gets the object but pays the (k+1)st bid.
    - Always best for i to bid b_i = v_i, true valuation. Why?
  - No-good---The goal is to give the object without taking away or giving money.

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  - No good: The goal is to give the object without taking away or giving money.

Second-price auction

Suppose your valuation of a good is \$100.

Let b_j be the highest bid other than yours.

If $b_j > 100$, say 109, . . .

If $b_j < 100$, say 96, . . .

"SIMPLE" MECHANISMS

- But the (k+1)st-price auction is useful for constructing a mechanism that solves the problem.
- E.g., Olszewski's mechanism (2003) uses a 2nd-price auction, modified by adding an extra payment from the planner:
  - If $b_1 > b_2$, then

    $u_1 = v_1 - b_2 + (b_2 - \delta) = v_1 - \delta$

    $u_2 = 0 + (b_1 - \delta) = b_1 - \delta$
  - A bit strange? More later.

NOW,
3000 YEARS AFTER
SOLOMON . . .

MIHARA'S MECHANISM

- Stage 1
  - Each agent either claims the object or not.
    - If at most k agents claim, they get the object.
    - Otherwise, go to Stage 2.
- Stage 2
  - The (k+1)st-price auction with entry fees δ

That's it!

HOW IT WILL WORK

- Each agent i bids $b_i = v_i$ in Stage 2.
- $i \in H$ claims the object since she enjoys a surplus of
  - $v_i > 0$ (if an auction not held) or
  - $v_i - b(k+1) - \delta > 0$ (if held),

  where $b(k+1)$ is the $(k+1)$st highest bid (by someone not in H).
- $j \notin H$ does not claim it since she has to pay δ if she claims it.

COMPARISON

OLSZEWSKI'S MECHANISM

- Each i bids $b_i = v_i$ in Stage 2.
- Given that, Stage 1 payoffs (assuming $b_1 > b_2$) are:

	hers	mine
hers	$v_1 - \delta$, $b_1 - \delta$	0, v_2
mine	v_1, 0	0, 0

BRIBE FROM 2 TO 1

	hers	mine
hers	$v_1 - \delta + t, b_1 - \delta - t$	$0, v_2$
mine	$v_1, 0$	$0, 0$

- Suppose $v_1 = 100, v_2 = 50, \delta = 20$.

  Euilibrium payoffs are (100, 0).
- 2 bids $b_2 = 0$ and gives $t = \$1000$ to 1. In return, 1 bids $b_1 = 2{,}000$.

  The payoffs are (1080, 980)!

- A pair of agents can gain in Olszewski's mechanism by bribing each other.
- No pair of agents can gain in this way in Mihara's mechanism.

REFERENCES

- H. R. Mihara, The second-price auction solves King Solomon's dilemma. Available from http://econpapers.repec.org/RAS/pmi193.htm
- H. Reiju Mihara’s website

  http://www5.atwiki.jp/reiju/