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Female Labor Supply, Fertility, and Heterogeneous Households

Megumi Mochida*

Abstract

We examine how female labor supply affects fertility in an overlapping-generations model, considering three different types of households exist in the economy. We show a case that an increase in the female labor participation rate increases the fertility rate. We also demonstrate that an increase in the social security tax rate and the amount of child allowance benefit can decrease the fertility rate in the economy.

Keywords : Overlapping generation, Fertility, Child Allowance, Pay-as-you-go social security.

JEL : D 91, H 55, J 13.

1 Introduction

We observe the positive relationship between female labor supply and total fertility in OECD countries, recently. The total fertility rate has been slightly increasing in Japan since 2005 around the same time that the female labor supply is also increasing.

This paper considers an overlapping-generations model in which young-working

＊ Fuculty of Economics, Kagawa University, 2-1 Saiwaicho, Takamatsu, Kagawa 760-8523, Japan.

generation is composed of three different types of households: households in which both wife and husband work outside during raising children, households with children in which only the husband works outside, and households of two workers without children. In such economy, we examine how an increase in a proportion of female labor supply affects the fertility rate through social security systems.

There are many papers which consider social security systems and fertility. Becker and Barro (1988) and Zhang (1995) show that increasing social security benefits reduces the fertility rate considering the bequest motives. Zhang and Zhang (1998) demonstrate that with both PAYG-pension and gifts (transfer from children to parents), a higher social security tax leads to lower fertility rate and leads to higher per-capita growth rate. The relationship between female labor and fertility is also discussed, theoretically and empirically. Ahn and Mira (2002) show that with a panel of OECD aggregate fertility and labor market data between 1970 and 1995, along the cross-sectional dimension, the correlation between total fertility rate and female labor participation rate was negative and significant during the 1970's and up to the early 1980's, however, by the late 1980's the correlation had become positive and equally significant. Apps and Rees (2004) analyze the extent to which the positive relationship between female labor supply and fertility can be explained by public policy, in particular taxation and the system of child support. The results suggest that countries which have individual rather than joint taxation, and which support families through child care facilities rather than child payments, are likely to have both higher female labor supply and higher fertility. This paper presents a positive relationship between female labor participation rate and fertility rate, theoretically, considering heterogeneous households and social security policies.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 shows a case in which the increase of female labor force participation increases the fertility rate. Section 4 shows fertility effects of social securities. Concluding remarks appear in Section 5.

2 Model

We consider a small open economy populated by overlapping generations who live through three periods : childhood, young, and old. By the assumption of a small open economy, firms and households take a world interest rate as given. The interest rate is assumed to be constant over time. The government implements two social security systems ; Child-Allowance systems and PAYG-Pension systems.

The settings of the childhood and the old period are mostly common to all individuals in the same generation except for the amount of pension benefit and savings. However, in the young period, the individuals divide into *three* types of households with an exogenous probability. The first type of households is consisted of a worker who receives labor income, a homemaker who does not earn wage income, and some children. We call this type of households as *H-Parents*. The second type is a family with children where both parents are employed outside and earn labor income, that is, so called double-income-with-kids family. We call this type as *D-Parents*. The third type of households is married *Couples*, in which husband and wife are working, without bringing own children.(1)

2.1 Households

We refer to a generation spending as a worker or a homemaker in period t as generation t. The population of generation t is denoted by N_t. It is assumed that the same number of male, N_t^m, and female, N_t^f, exist in the economy. Thus, $N_t^m = N_t^f = \frac{1}{2}N_t$. To focus on the female labor participation, we assume that only χ part of the female young population works, and $(1-\chi)$ part of them is regarded as homemakers (housewives) who do not earn wage income by themselves, in

(1) Although we regard all young individuals as couples to avoid complicacy, the result of this paper is not changed if the member of *Couples* are not married and make independent living. At that time, the number of households is only changed.

contrast, all male young individuals work outside to earn wage income. Thus the number of young female population of *H-Parents* is $(1-\chi)N_t^f$. Among the female workers, χN_t^f, ϕ part of them is called as *Couples* who do not bring own children. The number of such females is written as $\phi\chi N_t^f$. $(1-\phi)$ part of the female employees, $(1-\phi)\chi N_t^f$, is called as *D-Parents* and raise some children during working outside. It is assumed that according to the distribution of female type in the young period the couples are formed, automatically. The number of households of *H-Parents* and *D-Parents* are, respectively, given by $(1-\chi)\frac{1}{2}N_t$, and $(1-\phi)\chi\frac{1}{2}N_t$. The amount of *Couples* in this economy is $\phi\chi\frac{1}{2}N_t$.

2.1.1 H-Parents

Each household of *H-Parents* derives utility both from material consumption in the second and third periods of life, and from the number of children. The life time utility function of the household of generation t is expressed as

$$u_t^H = \ln c_t^{yH} + p \ln c_{t+1}^{oH} + \sigma \ln n_t^H, \tag{1}$$

where c_t^{yH} and c_{t+1}^{oH} denote consumption in the second and third periods of life, respectively, n_t^H is the number of children, $p \in (0, 1)$ is the discount factor, which is common among the same generation, and $\sigma(>0)$ represents the degree of the preference for children.

The budget constraints of the member of generation t when young is given by

$$(1-\tau)w + Q_t n_t^H = c_t^{yH} + s_t^H + qn_t^H, \tag{2}$$

where τ is the rate of payroll tax for social security systems, w is the wage rate, Q_t is the amount of child allowances per child, s_t^H is the savings for consumption in the retirement period, and q is the rearing cost per child.

The budget constraint in the retirement period is written as

$$(1+r)s_t^H + T_{t+1}^H = c_{t+1}^{oH}, \tag{3}$$

where $(1+r)$ denotes the interest factor, T_{t+1}^H is the pension benefit paid in period $t+1$.

Combining (2) and (3) leads to the household's lifetime budget constraint:

$$c_t^{yH} + qn_t^H + \frac{c_{t+1}^{oH}}{(1+r)} = (1-\tau)w + Q_t n_t^H + \frac{T_{t+1}^H}{(1+r)}. \tag{4}$$

The households choose consumption in the second and third periods together with the number of children so as to maximize lifetime utility (Eq. 1) subject to the lifetime budget constraint (Eq. 4). The first-order conditions for the optimum are given by

$$\frac{1}{c_t^{yH}} = \frac{(1+r)}{c_{t+1}^{oH}}$$

and

$$\frac{(q-Q_t)}{c_t^{yH}} = \frac{\sigma}{n_t^H}.$$

Making use of the first-order conditions, we have

$$n_t^H = \frac{\sigma}{(q-Q_t)}\eta\left[(1-\tau)w + \frac{T_{t+1}^H}{(1+r)}\right] \tag{5}$$

and

$$s_t^H = p\eta\left[(1-\tau)w + \frac{T_{t+1}^H}{(1+r)}\right] - \frac{T_{t+1}^H}{(1+r)},$$

where $\eta \equiv \frac{1}{(1+p+\sigma)} \in (0,\ 1)$.

2.1.2 D-Parents

The setting of *D-Parents* is mostly the same as that of *H-Parents* except for wage income in the young, and the amount of pension benefit in the old period. The life time utility function of *D-Parents* of generation t is expressed as

$$u_t^D = \ln c_t^{yD} + p \ \ln c_{t+1}^{oD} + \sigma \ \ln n_t^D, \tag{6}$$

where c_t^{yD} and c_{t+1}^{oD} denote consumption in the second and third periods of life, respectively, n_t^D is the number of children, and the degree of σ is the same as that of *H-Parents*.

The budget constraints when young is given by

$$2(1-\tau)w + Q_t n_t^D = c_t^{yD} + s_t^D + q n_t^D. \tag{7}$$

As *D-Parents* receive dual income, their income is twice as the income of *H-Parents*. The saving is given by s_t^D.

The budget constraint in the retirement period is written as

$$(1+r)s_t^D + T_{t+1}^D = c_{t+1}^{oD}, \tag{8}$$

where T_{t+1}^D is the pension benefit which this type of households receive in period $t+1$.

Combing (7) and (8) leads to the household's lifetime budget constraint :

$$c_t^{yD} + q n_t^D + \frac{c_{t+1}^{oD}}{(1+r)} = 2(1-\tau)w + Q_t n_t^D + \frac{T_{t+1}^D}{(1+r)}. \tag{9}$$

The households choose consumption in the second and third periods together with the number of children so as to maximize lifetime utility (Eq. 6) subject to the lifetime budget constraint (Eq. 9). The first-order conditions for the optimum are given by

$$\frac{1}{c_t^{yD}} = \frac{p(1+r)}{c_{t+1}^{oD}}$$

and

$$\frac{(q-Q_t)}{c_t^{yD}} = \frac{\sigma}{n_t^D}.$$

Making use of the first-order conditions, we have

$$n_t^D = \frac{\sigma}{(q - Q_t)} \eta [2(1-\tau) w + \frac{T_{t+1}^D}{(1+r)}] \tag{10}$$

and

$$s_t^D = p\eta [2(1-\tau) w + \frac{T_{t+1}^D}{(1+r)}] - \frac{T_{t+1}^D}{(1+r)}.$$

2.1.3 Couples

Each household derives utility only from material consumption in the second and third periods of life. The life time utility function of a household of generation t is expressed as

$$u_t^C = \ln c_t^{yC} + p \ln c_{t+1}^{oC}, \tag{11}$$

where c_t^{yC} and c_{t+1}^{oC} denote consumption in the second and third periods of life, respectively.

The budget constraints when young is given by

$$2(1-\tau) w = c_t^{yC} + s_t^C, \tag{12}$$

where s_t^C is the savings for consumption in the retirement period.

The budget constraint in the retirement period is written as

$$(1+r) s_t^C + T_{t+1}^C = c_{t+1}^{oC}, \tag{13}$$

where T_{t+1}^C is the pension benefit paid in period $t+1$.

Combining (12) and (13) leads to the household's lifetime budget constraint:

$$c_t^{yC} + \frac{c_{t+1}^{oC}}{(1+r)} = 2(1-\tau) w + \frac{T_{t+1}^C}{(1+r)}. \tag{14}$$

The households choose consumption in the second and third periods so as to maximize lifetime utility (Eq. 11) subject to the lifetime budget constraint (Eq. 14). The first-order conditions for the optimum are given by

$$\frac{1}{c_t^{yC}}=\frac{p(1+r)}{c_{t+1}^{oC}}.$$

Making use of the first-order conditions and (14) we have

$$s_t^{yC}=\frac{p}{(1+p)}[2(1-\tau)w+\frac{T_{t+1}^C}{(1+r)}]-\frac{T_{t+1}^C}{(1+r)}.$$

2.2 Production

Competitive firms produce a single final good by employing both physical capital and labor input. The aggregate production function at time t is given by $Y_t=F(K_t, L_t)=K_t^{\alpha}L_t^{1-\alpha}$, where Y_t, K_t, L_t, and $\alpha\in(0, 1)$ respectively denote the aggregate output, the physical capital, the aggregate labor, and the share of physical capital. The aggregate labor, L_t, is

$$\begin{aligned} L_t &= N_t^m+\chi N_t^f \\ &= \frac{1}{2}N_t+\chi\frac{1}{2}N_t \\ &= (1+\chi)\frac{1}{2}N_t. \end{aligned} \tag{15}$$

Firms hire physical capital and labor inputs up to the point where their marginal products equal their factor prices :

$$\begin{aligned} (1+r) &= \frac{\partial Y_t}{\partial K_t}=\alpha K_t^{\alpha-1}L_t^{1-\alpha}, \\ w &= \frac{\partial Y_t}{\partial L_t}=(1-\alpha)K_t^{\alpha}L_t^{-\alpha}, \end{aligned}$$

where r is the world rate of interest.

2.3 The government

This economy has two social security systems ; Child-Allowance and PAY-G Pension systems. The government levies a tax $\tau\in[0, 1]$ on wages of young workers and redistributes $\mu \in [0, 1]$ part of the revenue to those young child-rearing households as child allowances, and $(1-\mu)$ part of it to retired workers as PAYG-public pensions. In this paper, we assume that the pension benefit is only allocated to retired workers, so homemakers who had not paid their contribution in the young period are not received own pension benefit in the old period.(2) The marginal contribution rate τ is assumed to be constant over time and common to all workers.

The value of allowance per child at time t is described as Q_t, which is adjusted to keep the government's budget constraint. The budget constraint of the child-allowance policy at time t is given by

$$\mu\tau w L_t = Q_t N_{t+1}. \tag{16}$$

The population of generation $t+1$ is expressed as

$$N_{t+1} = n_t^H (1-\chi)\frac{1}{2}N_t + n_t^D (1-\phi)\chi\frac{1}{2}N_t. \tag{17}$$

Using (15)-(17), the value of child-allowance per child is determined as

$$Q_t = \frac{\mu\tau w L_t}{N_{t+1}} = \frac{\mu\tau w(1+\chi)}{n_t^H(1-\chi)+n_t^D(1-\phi)\chi}. \tag{18}$$

The amount of pension benefit per retired workers at time t is described as T_t and adjusted to keep the government's budget constraint. The budget constraint of the PAYG-pension system at time t is given by

$$(1-\mu)\tau w L_t = T_t L_{t-1}.$$

The pension benefit per retired worker at time t is determined as

$$T_t = \frac{(1-\mu)\tau w L_t}{L_{t-1}} = \frac{(1-\mu)\tau w N_t}{N_{t-1}}. \tag{19}$$

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(2) Even if all old individuals including homemakers receive the same amount of pension benefit, the results of this paper does not change significantly.

The pension benefits which each type of the households of generation t receives at time $t+1$ are as follows.

$$T^H_{t+1} = T_{t+1},$$
$$T^D_{t+1} = 2T_{t+1},$$
$$T^C_{t+1} = 2T_{t+1}.$$

2.4 Equilibrium

We assume that individuals have perfect foresight not only into future interest rates and social security benefits but also the number of children in the next generation.

Inserting government's budget constraints (18), (19) into households' equilibria (5), (10), and using (17), we can get the numbers of children those *H-Parents* and *D-Parents* decide to have in equilibrium, respectively, as follows.

$$n^H_t = \frac{2w(1+r)[\mu\tau(1+\chi)+\sigma\eta(1-\tau)\{1+\chi(1-2\phi)\}]}{[2q(1+r)-\sigma\eta(1-\mu)\tau w\{1+\chi(1-2\phi)\}][1+\chi(1-2\phi)\}}, \tag{20}$$

$$n^D_t = 2n^H_t = \frac{4w(1+r)[\mu\tau(1+\chi)+\sigma\eta(1-\tau)\{1+\chi(1-2\phi)\}]}{[2q(1+r)-\sigma\eta(1-\mu)\tau w\{1+\chi(1-2\phi)\}][1+\chi(1-2\phi)\}}. \tag{21}$$

To make sure that n^H_t and n^D_t are positive, we assume that the denominators of (20) and thus (21) are positive.(3) That is,

$$q > \frac{\sigma\eta(1-\mu)\tau w\{1+\chi(1-2\phi)\}}{2(1+r)} \equiv q_0.$$

Combining (20) and (21) leads to the population growth rate in equilibrium, as follows.

(3) We can confirm the numerators of (20) and thus (21) are positive, easily.

$$\{1+\chi(1-2\phi)\} \geq 0 \Leftrightarrow \phi \leq \frac{1+\chi}{2\chi} = \frac{1}{2\chi} + \frac{1}{2} \equiv \tilde{\phi}$$

Under $\chi \in (0,\ 1)$, then $\tilde{\phi} > 1$. By the setting of $\phi \in (0,\ 1)$, the sign of $\{1+\chi(1-2\phi)\}$ is always positive.

$$n_t = \frac{N_{t+1}}{N_t} = \frac{w(1+r)[\mu\tau(1+\chi)+\sigma\eta(1-\tau)\{1+\chi(1-2\phi)\}]}{[2q(1+r)-\sigma\eta(1-\mu)\tau w\{1+\chi(1-2\phi)\}]} \equiv n^*.$$

They are constant over time.

3 Female labor participation and Fertility

In this section, we examine the effect of female labor force participation rate on the fertility rate in the economy. First of all, we assume that more than half of the female working population has their children, as follows.(4)

Assumption 1.

$$\phi \leq \frac{1}{2}.$$

Under Assumption 1., we have

Proposition 1.

An increase in the female labor force participation rate increases the total fertility rate.

Proof. The partial derivative of fertility with respect to χ is given by

$$sign\left(\frac{\partial n^*}{\partial \chi}\right)$$

$$= sign(q(1+r)[\mu\tau+\sigma\eta(1-\tau)(1-2\phi)]-\sigma\eta(1-\mu)\mu\tau^2 w\phi). \tag{22}$$

(4) If less than half of the female labor have children, that is, $\phi > \frac{1}{2}$, the result of Proposition 1. changes, as follows.

Under $\tau > \hat{\tau}$, then $\frac{\partial n^*}{\partial \chi} > 0$, under $\tau \leq \hat{\tau}$, then $\frac{\partial n^*}{\partial \chi} < 0$, where $\hat{\tau} \equiv \frac{-\sigma\eta(1-2\phi)}{\mu-\sigma\eta(1-2\phi)} \in (0, 1)$.

When $\tau > \hat{\tau}$, it is the same way as Proposition 1., under Assumption 1.

The sign of $\frac{\partial n^*}{\partial \chi}$ is positive if and only if $q > \frac{\sigma\eta(1-\mu)\mu\tau^2 w\phi}{(1+r)[\mu\tau+\sigma\eta(1-\tau)(1-2\phi)]} \equiv q_1$. Under Assumption 1., we always have $q_0 \geq q_1$. Thus, the sign of $\frac{\partial n^*}{\partial \chi}$ is always positive.

Q. E. D.

There are two ways which the fertility rate increases as the female labor force participation rate increases. First of all, when female labor participation rate increases, the child allowance per child increases as the tax revenue increases. Thus, the number of children which both *H-Parents* and *D-Parents* choose to have increases. Secondly, as richer *D-Parents* have more children than that of *H-Parents* have, the increase of the female labor participation rate raises the number of *D-Parents* leads to higher fertility rate.

4 Social security systems and Fertility

In this section, we examine the effects of the social security systems on the fertility rate.

The following propositions summarize effects of the increase of the social security tax rate and the volume of child allowance benefit on the population growth rates.

Proposition 2.

(1) When $\mu \geq \hat{\mu}$, an increase in the social security tax rate increases the fertility rate.

(2) When $\mu < \hat{\mu}$, an increase in the social security tax rate increases (decreases) the fertility rate, if $q < (>) q_2$.

Proof. The partial derivative of fertility with respect to τ is given by

$$sign\left(\frac{\partial n}{\partial \tau}\right)$$

$$= sign(2q(1+r)[\mu(1+\chi)-\sigma\eta\{1+\chi(1-2\phi)\}]+\sigma^2\eta^2\{1+\chi(1-2\phi)\}^2(1-\mu)w). \tag{23}$$

If $[\mu(1+\chi)-\sigma\eta\{1+\chi(1-2\phi)\}]\geq 0$, that is, $\mu \geq \dfrac{\sigma\eta\{1+\chi(1-2\phi)\}}{(1+\chi)} \equiv \hat{\mu} \in (0, 1)$, then $\dfrac{\partial n}{\partial \tau} > 0$.

If $[\mu(1+\chi)-\sigma\eta\{1+\chi(1-2\phi)\}]< 0$, that is, $\mu < \hat{\mu}$, $\dfrac{\partial n}{\partial \tau} \geq (\leq) 0$ if and only if $q \leq (\geq) \dfrac{-\sigma^2\eta^2\{1+\chi(1-2\phi)\}^2(1-\mu)w}{2(1+r)[\mu(1+\chi)-\sigma\eta\{1+\chi(1-2\phi)\}]} \equiv q_2$.

Q. E. D.

When $\mu < \hat{\mu}$, pension benefit per retired worker is relatively larger and child allowance per child is smaller. At that time, if the rearing cost per child is comparatively large as $q > q_2$, the negative effect of the decrease in the disposable income dominates the positive effect of the increase of the social security benefit. Thus, the increase in the social security tax burden can decrease the fertility rate in the economy.

Proposition 3.

An increase of the allocation of child-allowance policy increases (decreases) the fertility rate, if $q > (<) q_3$.

Proof. We can show that the condition that the partial derivative of fertility with respect to μ is positive, as follows.

$$sign\left(\frac{\partial n}{\partial \mu}\right)$$

$$= sign(-\sigma\eta w\{1+\chi(1-2\phi)\}[\tau(1+\chi)+\sigma\eta(1-\tau)\{1+\chi(1-2\phi)\}]+2q(1+r)(1+\chi)) \geq 0$$

$$\Leftrightarrow q \geq \frac{\sigma\eta w\{1+\chi(1-2\phi)\}[\tau(1+\chi)+\sigma\eta(1-\tau)\{1+\chi(1-2\phi)\}]}{2(1+r)(1+\chi)} \equiv q_3.$$

Q. E. D.

The smaller the rearing cost is, the larger the fertility rate in the economy is. In that case, when the government increases the child allowances while decreasing the pension benefit, the households with children increases the saving in preparation for the retired life, and decreases the number of children those they choose to have.

5 Concluding remarks

This paper has presented an economy in which young working generation is composed of three-type households. In such economy, we showed a case that an increase in the female labor force participation rate increases the fertility rate. We also demonstrated that an increase in the volume of social security and even child allowance benefit may decrease the fertility rate depending on the child rearing cost in the economy.

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