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Fertility behavior in an aging economy

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Abstract

This paper considers an overlapping-generations model in which young-working generation is composed of two different types; *Parents*, who bring up own children, and *Adults*, who do not have own children. We examine how an increase in a proportion of *Adults* affects the fertility and growth rates. We also demonstrate that the presence of PAYG-pension systems increases the fertility rate, if a proportion of *Parents* is large enough. If a proportion of *Adults* is sufficiently large, the existence of pension systems decreases the fertility rate.

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1 Introduction

Advanced countries are aging rapidly. In such countries, we also observe an increase in population who do not have own children, besides higher life-expectancy and declining fertility rate.[1]

This paper considers an overlapping-generations model in which young-working generation is composed of two different types; *Parents*, who bring up own children, and *Adults*, who do not have own children.[2] The government implements two social security systems; PAYG-pension and Child-allowance systems. Young-workers, in other words, consist of two kinds of tax-payers; *Parents*, who pay smaller pay-roll tax and receive larger benefit than *Adults*,

[1]The total fertility rate in Japan is 1.46 (2015). This small fertility rate is regarded as stemming from increasing of unmarried person. For example, in 2010, unmarried male population at the age of between 30 and 34 and between 35 and 39 is, respectively, 47.3%, and 35.6%. Unmarried female population at the age of between 30 and 34 and between 35 and 39 is also low as 34.5%, and 23.1%. Thus, the existence of people who do not reproduce the next generation is significant when we consider policy effects, especially, for children and families.

[2]Our model supposes that *Adults'* labor supply is exogenously fixed, but, on the other hand, *Parents'* labor supply is a choice variable. Therefore, following a model by Abio et al. (2004), we can regard *Adults* in here as "male labor" and *Parents* as "female (-mother) labor", if ignoring a proportion of house-wife mothers.

and *Adults*, who pay larger pay-roll tax and receive smaller benefit than *Parents*. Under PAYG-pension is operated, *Adults* in old age are supported economically by the present children. We examine how an increase in a proportion of *Adults* affects the fertility and growth rates through public policies.

The increase in *Adults'* population accelerates declining fertility rates. Diminishing young population and a heavier economic burden on the young-generation threaten the feasibility of current social security systems in aging economies. We examine whether the presence of PAYG-pension systems has positive effects on the fertility rate or not. It is shown that the introduction of PAYG-pension systems increases the fertility rate, if a proportion of *Parents* is large enough. If a proportion of *Adults* those who receive smaller benefit than families is sufficiently large, the existence of pension systems decreases the fertility rate.

Becker and Barro (1988) and Zhang (1995) shows that increasing social security benefits reduces the fertility rate considering the bequest motives. Zhang and Zhang (1998) demonstrate that with both PAYG-pension and gifts (transfer from children to parents), a higher social security tax leads to lower fertility rate and leads to higher per-capita growth rate. Our paper analyzes how demographic change and social security policies affect the fertility and economic growth rates, considering two different taxpayers in the same generation.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 explains how an increase in *Adults'* population affects fertility and per-capita growth rates. Section 4 shows fertility effects of pension systems. Concluding remarks appear in Section 5.

2 The model

We consider an overlapping-generations model of endogenous growth, incorporating an uncertain lifetime. The life of a representative individual is divided into three periods: a childhood and a young-working period, both with fixed durations, and a retirement period of uncertain length. For simplicity, we assume that the individual is either alive or dead at the beginning of the third period. The probability that the individual is alive at the beginning of the retirement period is denoted as $p \in (0, 1]$, which is known and common to individuals in the all generation. Assuming that insurance companies are risk-neutral and that private annuities markets are competitive, the expected rate of return to savings is $(1+\rho_{t+1}) = \frac{(1+r_{t+1})}{p}$, where $(1+r_{t+1})$ is the return of direct holdings of capital. In the absence of bequest motives, individuals are willing to invest their assets in such insurance.[3]

The settings of the childhood and the retirement period are common to all individuals in the same generation; for example, human capital level which they accumulate in the first period, survival probability to the third period, and the amount of pension benefit that living individuals receive in the retirement period. However, in the young-working period, the individuals divide into two kinds of workers with an exogenous probability. $(1-\eta)$ part of the young generation is called by *Parents*, who reproduce some children. *Parents* allocate their time to raising children as well as working to receive labor income. During the time for bringing-up, *Parents* are not able to work to receive wage income. Therefore, the income during the bringing-up is opportunity cost of having own children. To alleviate the economic burden on *Parents*, the government provide them with child allowances according to the number of children those *Parents* have. On the other had, η part of the young-working generation is called by *Adults*, who do not have own children in their whole life. *Adults* spend their time in working to receive wage income in the young-working period.

[3]This is a simplified version of Blanchard's (1985) model.

2.1 Households

Each individual who is born at time t, and called generation $t+1$, accumulates human capital through education by her/his parent at time t. Given parents' human capital, h_t, when a parent allocates time e_t toward parental-teaching for one child, the child accumulates human capital h_{t+1} according to

$$h_{t+1} = \theta e_t^{\gamma} h_t,$$

where $\theta > 0$ and $\gamma \in (0, 1)$.

2.1.1 Parents

Parents are endowed with one divisible unit of time in their young periods, reproduce asexually, and allocate their time toward labor and raising children. To raise a child, they are compelled to spend their time in child-rearing and parental-teaching, which helps their children to accumulate human capital.[4] They receive wage income, which is taxed away, and allowances according to the number of children in the end of their young periods. They consume part of their income and invest the rest of their income in annuities. Subsequently, living individuals obtain the principal and interest from their

[4]In this model, as in Eckstein and Zilcha (1994), the decision of education time is based on parental altruism.

annuities and consume them with their pension benefits after retirement.

The endowment time in the young period is normalized to unity. The time constraint of generation t is then given by

$$1 = l_t^P + n_t(q + e_t),$$

where l_t^P, n_t, and q respectively denote the labor time of *Parents*, the number of children, and the rearing time per child.

The budget constraints of a member of generation t when young and retired are given, respectively, by

$$(1 - \tau - \omega)w_t h_t l_t^P + Q_t n_t = c_t^{yP} + a_t^P$$

and

$$(1 + \rho_{t+1})a_t^P + T_{t+1} = c_{t+1}^{oP},$$

where c_t^{yP} is the consumption when young, a_t^P is the annuity of *Parents*, T_{t+1} is the pension benefit, and c_{t+1}^{oP} represents post-retirement consumption.

All *Parents* have identical preferences. The utility function of generation $t(\geq 0)$ is represented by[5]

$$u_t^P = \ln c_t^{yP} + p\ln c_{t+1}^{oP} + \sigma \ln n_t h_{t+1}.$$

[5]With an uncertain lifetime, $p \in (0, 1]$, this utility form is employed by Pecchenino and Pollard (1997), Pecchenino and Utendorf (1999), and Yakita (2001), among others.

The parameter $\sigma \in (0,1)$ can be interpreted as the degree of preference over "total human capital" of their children.

By solving individuals' optimization problems, the optimal values are given by:

$$n_t = \frac{\sigma}{(1+p+\sigma)} \frac{(1-\gamma)}{\{q(1-\tau-\omega)w_t h_t - Q_t\}} I_t,$$

$$l_t^P = 1 - \frac{\sigma}{(1+p+\sigma)} \frac{\{q(1-\tau-\omega)w_t h_t - \gamma Q_t\}}{\{q(1-\tau-\omega)w_t h_t - Q_t\}} \frac{1}{(1-\tau-\omega)w_t h_t} I_t,$$

$$e_t = \frac{\gamma\{q(1-\tau-\omega)w_t h_t - Q_t\}}{(1-\gamma)(1-\tau-\omega)w_t h_t},$$

$$c_t^{yP} = \frac{1}{(1+p+\sigma)} I_t,$$

$$c_{t+1}^{oP} = (1+\rho_{t+1}) \frac{p}{(1+p+\sigma)} I_t,$$

$$a_t^P = \frac{p}{(1+p+\sigma)} I_t - \frac{T_{t+1}}{(1+\rho_{t+1})},$$

where $I_t \equiv (1-\tau-\omega)w_t h_t + \frac{T_{t+1}}{(1+\rho_{t+1})}$.

2.1.2 Adults

Adults spend their all time in working in their young periods. Thus, *Adults'* labor supply is exogenously fixed by $l_t^A = 1$. They receive wage income, which is taxed away, and consume part of the income, and invest the rest of their income in annuities. Subsequently, living individuals obtain the principal and interest from their annuities and consume them with their pension benefits after retirement.

The budget constraints of a member of generation t when young and retired are given, respectively, by

$$(1-\tau-\omega)w_t h_t = c_t^{yA} + a_t^A,$$

and

$$(1+\rho_{t+1})a_t^A + T_{t+1} = c_{t+1}^{oA},$$

where c_t^{yA} is the consumption when young, a_t^A is the annuity of *Adults*, T_{t+1} is the pension benefit, and c_{t+1}^{oA} represents post-retirement consumption.

All *Adults* have identical preferences. The utility function of generation $t(\geq 0)$ is represented by

$$u_t^A = \text{ln}c_t^{yA} + p\text{ln}c_{t+1}^{oA}.$$

By solving individuals' optimization problems, the optimal values are given by:

$$c_t^{yA} = \frac{1}{(1+p)} I_t,$$

$$c^{oA}_{t+1} = (1+\rho_{t+1})\frac{p}{(1+p)}I_t,$$

$$a^A_t = \frac{p}{(1+p)}I_t - \frac{T_{t+1}}{(1+\rho_{t+1})},$$

where $I_t \equiv (1-\tau-\omega)w_t h_t + \frac{T_{t+1}}{(1+\rho_{t+1})}$ is the same as in 2.1.1.

2.2 Production

Competitive firms produce a single final good by employing both physical capital and effective labor input. The aggregate production function at time t is given by $Y_t = F(K_t, h_t L_t) = K^\alpha_t (h_t L_t)^{1-\alpha}$, where Y_t, K_t, L_t, and $\alpha \in (0,1)$ respectively denote the aggregate output, the physical capital which fully depreciates in the production process, the aggregate labor, and the share of physical capital. The aggregate labor, L_t is

$$L_t = l^P_t N^P_t + l^A_t N^A_t$$

$$= l^P_t (1-\eta)N_t + \eta Nt.$$

The production function, in intensive form, can be expressed as

$$\tilde{y}_t = \tilde{k^\alpha_t},$$

where $\tilde{y}_t = \frac{Y_t}{h_t L_t}$ is the output per effective-labor and $\tilde{k}_t = \frac{K_t}{h_t L_t}$ is the physical capital per effective-labor.

The factor markets are presumed to be perfectly competitive. Therefore, the firms take factor prices as given. Firms hire labor inputs and physical capital up to the point where their marginal products equal their factor prices:

$$w_t = (1-\alpha)\tilde{k_t}^{\alpha},$$

$$(1+r_t) = \alpha\tilde{k_t}^{\alpha-1}.$$

2.3 The government

The government implements two social security systems; PAYG-Pension systems and Child-Allowance systems. For pension systems, the government levies a tax $\tau \in [0,1)$ on wages of young individuals and redistributes to retired individuals. The payment per retired person at time t is described as T_t. The tax rate, τ, and pension benefit, T_t, are common to both *Parents* and *Adults*. Therefore, the pension systems in this model play not only a role of intergenerational redistribution, but also a role of intra-generational redistribution. That is, if *Parents* are alive at the third period, they receive the same amount of pension benefit as *Adults*, despite the smaller contribution that they have paid in the working period.[6] The government levies a tax $\omega \in [0, 1)$ on wages of all young workers and redistributes to *Parents* as child allowances. The value of the allowance per child at time t is described as Q_t. This child-allowance systems play a role of intra-generational redistribution from *Adults*, who work longer time and receive higher income, to *Parents*, who work shorter time and receive lower income.

The budget constraint of the PAYG-pension system at time t is given by

$$\tau w_t h_t L_t = T_t p N_{t-1},$$

where $L_t = l_t^P N_t^P + l_t^A N_t^A$, and $pN_{t-1} = pN_{t-1}^P + pN_{t-1}^A$.

The pension benefit per retired is determined as

$$T_t = \frac{\tau w_t h_t L_t}{p N_{t-1}}.$$

The budget constraint of the child-allowance policy at time t is given by

$$\omega w_t h_t L_t = Q_t n_t N_t^P,$$

where $N_t^P = (1 - \eta) N_t$.

[6]Under the different pension systems, for example, funded pension systems or fertility-related pensions, we may have another results.

The value of child-allowance per child is determined as

$$Q_t = \frac{\omega w_t h_t L_t}{n_t(1-\eta)N_t}$$

$$= \frac{\omega w_t h_t\{l_t^P(1-\eta)+\eta\}}{n_t(1-\eta)}.$$

As for the efficiency of education, we assume that the efficiency rate of education is sufficiently small to confirm the following discussion, as follows.[7]

Assumption 1.

$$\gamma < \frac{(1-\tau-\omega)}{(1-\tau)} \in (0,1).$$

2.4 Equilibrium

Using capital market-clearing conditions, $K_{t+1} = a_t^P N_t^P + a_t^A N_t^A$, we can get the values at equilibrium as follows. The number of children those *Parents* decide to have, n^*, and education time per-child, e^*, are constant over time:

$$n_t = \frac{[\{(1-\gamma)(1-\tau)-\omega\}(1-\eta)\kappa\sigma(1+\frac{(1-\alpha)}{\alpha}\tau)+\omega\{1+(1-\Sigma)\frac{(1-\alpha)}{\alpha}\tau\}]}{q(1-\tau)(1-\eta)\{1+(1-\Sigma)\frac{(1-\alpha)}{\alpha}\tau\}}$$

$$\equiv n^*,$$

[7]When this assumption is not fold, we have to set another assumption to get positive values of e^*.

$$e_t = \frac{\gamma\kappa\sigma(1+\frac{(1-\alpha)}{\alpha}\tau)}{n^*\{1+(1-\Sigma)\frac{(1-\alpha)}{\alpha}\tau\}}\frac{\{(1-\gamma)(1-\tau)-\omega\}}{\{(1-\gamma)(1-\tau)-\gamma\omega\}} + \frac{q\gamma\omega}{\{(1-\gamma)(1-\tau)-\gamma\omega\}}$$

$$\equiv e^*,$$

where $\kappa \equiv \frac{1}{(1+p+\sigma)}$, $\Sigma = p\kappa + \frac{\sigma p\kappa}{(1+p)}\eta$, and $(1-\Sigma) = \frac{1}{(1+p)} + \frac{\sigma p \kappa}{(1+p)}(1-\eta)$.

The population growth rate, g_N, is explained as

$$g_N = \frac{N_{t+1}}{N_t} = \frac{n^* N_t^P}{N_t} = \frac{n^*(1-\eta)N_t}{N_t} = n^*(1-\eta).$$

The growth rate of per-capita output at time t is

$$(1+g_t) \equiv \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}} = (\frac{\tilde{k}_{t+1}}{\tilde{k}_t})^\alpha \frac{h_{t+1}}{h_t}.$$

The capital per effective labor becomes $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$ in the steady state.[8] Consequently, the per-capita growth rate in the balanced-growth path depends only on the parental-teaching time. It is given by

$$(1+g^*) = \theta(e^*)^\gamma.$$

[8]The dynamics is always stable due to assumption of $\alpha \in (0,1)$.

3 Demographic effects

Let's see the demographic effects on the equilibria when *Adults'* population increases.

Proposition 1.

The increase in a proportion of Adults leads to larger number of children those Parents decide to have, and smaller education time per child. And it also decreases the per-capita growth rate.

Proof. The following equation represents the effect on the number of children those *Parents* decide to have.

$$sign(\frac{\partial n^*}{\partial \eta})$$

$$= sign(\{(1-\gamma)(1-\tau)-\omega\}\frac{p\sigma^2\kappa^2}{(1+p)}\frac{(1+\frac{(1-\alpha)}{\alpha}\tau)\frac{(1-\alpha)}{\alpha}\tau}{\{1+(1-\Sigma)\frac{(1-\alpha)}{\alpha}\tau\}^2}+\frac{\omega}{(1-\eta)^2}) > 0.$$

This sign is satisfied with any $\eta \in [0,1)$, under the Assumption 1. [9]

[9]The demographic effect on the population growth rate, g_N, is contrarily negative, as

$$sign(\frac{\partial g_N}{\partial \eta}) = sign(-\frac{1}{(1+p)}\frac{(1-\alpha)}{\alpha}\tau - 1) < 0.$$

Secondly, the effect on the education time per child is given by

$$sign(\frac{\partial e^*}{\partial \eta}) = sign(-\frac{1}{(1+p)}\frac{(1-\alpha)}{\alpha}\tau - 1) < 0.$$

This sign is satisfied with any η.

Therefore, the effect on the per-capita growth rate in the balanced-growth path depends on the per-child education time; it is given by

$$sign(\frac{\partial(1+g^*)}{\partial \eta}) = sign(\frac{\partial e^*}{\partial \eta}) < 0.$$

Q.E.D.

These results are explained as follows. An increase in a proportion of *Adults* promotes intra-generational transfer from *Adults* to *Parents*. Thus, *Parents* with larger child-allowance benefits increase the number of children those they have. As for negative effect on the per-capita education time, the method in our model that the government provide *Parents* with child allowances according to the number of children those they are bringing up has strong incentive to increase the number of children, decreasing the education level per child.

4 Fertility and PAYG-pension systems

The following propositions summarize effects of the presence of PAYG-pension systems on the population growth rates in the economy.[10]

Proposition 2.

Under $\omega > \tilde{\omega}$, when a proportion of Adults is more than (less than) $\hat{\eta}$, that is, a proportion of Parents is less than (more than) $(1 - \hat{\eta})$, the presence of PAYG-pension systems decreases (increases) the population growth rate.

Proposition 3.

Under $\omega \leq \tilde{\omega}$, the presence of PAYG-pension systems decreases the population growth rate.

[10] These results are the same as the effects on the number of children those *Parents* decide to have, n^*.

Proof. The following equation represents the effect on the population growth rate.

$$sign(\frac{\partial g_N}{\partial \tau}|_{\tau=0})$$
$$= sign(-(1-\gamma-\omega)\frac{\sigma^2 p\kappa}{(1+p)}\frac{(1-\alpha)}{\alpha}\eta^2$$
$$-[(1-\gamma-\omega)p\kappa\frac{(1-\alpha)}{\alpha}(1-\frac{\sigma}{(1+p)})+(1-\omega)]\sigma\eta$$
$$+[(1-\gamma-\omega)\sigma p\kappa\frac{(1-\alpha)}{\alpha}-(1+p)(1-\omega)]) \equiv g(\eta).$$

This function $g(\eta)$ is a quadratic function of η. The graph is convex upward and the value of $g(\eta = 1)$ is negative.[11]

The condition by which "the value of $g(\eta = 0)$ is positive" allows the graph of $g(\eta)$ to intersect the x-axis once in $\eta \in (0,1)$. When the value of $g(\eta = 0)$ is positive, ω is satisfied with the following inequality:

$$\omega > \frac{(1-\gamma)\sigma p\kappa\frac{(1-\alpha)}{\alpha}-(1+p)}{\sigma p\kappa\frac{(1-\alpha)}{\alpha}-(1+p)} \equiv \tilde{\omega}.$$

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$$g(\eta = 1) = -(1-\omega)(1+p+\sigma) < 0$$

When $\omega > \tilde{\omega}$, the intersection is a larger solution of $g(\eta) = 0$. We shall denote the solution by $\hat{\eta}$.[12]

When $\omega \leq \tilde{\omega}$, the value of $g(\eta)$ is negative for any $\eta \in (0, 1)$.

Q.E.D.

When η is larger, the young-working generation is composed of larger *Adults* and smaller *Parents*. When *Parents* receive larger child-allowance benefit than the case of smaller η, the introduction of PAYG-pension systems rather decrease the expected life-time income to decrease the number of children those *Parents* decide to have. It is because the PAYG-pension systems has weaker intra-generational redistribution effects from *Adults* to *Parents* than the child-allowance systems which has a direct transfer mechanism. At that time, if the government introduces the pension systems, the negative "price effect", by which child-rearing cost increases (although the cost is smaller than without the child allowances) through smaller child allowances dominates the positive "income effect" by larger expected income through receiving pension benefit.

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$$\hat{\eta} \equiv \frac{-B + \sqrt{B^2 + 4AC}}{2A},$$

where $A \equiv (1 - \gamma - \omega)\frac{\sigma^2 p\kappa}{(1+p)}\frac{(1-\alpha)}{\alpha}$, $B \equiv [(1 - \gamma - \omega)p\kappa\frac{(1-\alpha)}{\alpha}(1 - \frac{\sigma}{(1+p)}) + (1 - \omega)]\sigma$, and $C \equiv (1 - \gamma - \omega)\sigma p\kappa\frac{(1-\alpha)}{\alpha} - (1 + p)(1 - \omega)$.

5 Concluding remarks

When we consider the effects of governmental policies and social securities on the economy, it may be important to take account of both household diversities and the distribution, just like real economy. Introducing the difference in the household behavior in point of having children, this paper has examined how both a shift of the proportion of households with/without own children and an introducing PAYG-pension system affect the fertility and growth rates in the economy. Introducing such a difference in the household behavior into the macroeconomic model is our main contribution.

It has long been discussed whether social security systems increase the fertility rate or not. By considering the heterogeneous households, our paper showed that fertility effects of PAYG-pension systems depends on the magnitude of contribution rates and the composition ratio of households with/without own children in the economy.

There are obviously various households in our economy. Our results show that it is important that we take account of the family composition when we discuss the social security effects in the modern economy.

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