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Education Subsidies and Heterogeneous Households

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Abstract

We examine how an introduction of education subsidies affects growth rates, incorporating an uncertain lifetime and heterogeneous households. We show that there is the education tax rate which maximizes the growth rate. We also demonstrate that the effect of introduction of the subsidies on the growth rate, depending on the level of education tax rate and the proportion of households that do not raise children in the economy.

Keywords: Education subsidies; Uncertain lifetime; Heterogeneous households.

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1 Introduction

Human capital accumulation through education is recognized as one of the most important engines of economic growth. Therefore, it may be significant for the government to supply education systems effectively in order to enhance the human capital level and the growth rate in the economy. When the government plan education policies, it is also important to consider household behavior because, not surprisingly, human capital level of children who become workers to pay labor taxes in future is partly determined by their parents' decision. In this paper, we examine effects of education subsidies to complement parental educational investment, considering aging effects and the difference of family configuration.

Many researchers, such as Azariadis and Drazen (1990), Stokey (1991), and Glomm and Ravikumar (1992), among others, consider the human capital accumulation and growth using the overlapping generations' model. [1]

Under the setting that the governmental education policy helps human capital accumulation, the effectiveness of education subsidies has been widely discussed by Zhang and Casagrande (1998), Kaganovich and Zilcha (1999), Caucutt and Kumar (2003), Wigger (2004), and Rojas (2004), among others.

Kaganovich and Zilcha (1999) show that the introduction of education subsidies can crowd out parental private investment and then has a negative effect on the growth rate. Our motivation is to determine the effectiveness of education subsidies in aging economies where parents decide how much invest in their children's education. Incorporating an uncertain lifetime and the difference of family configuration to a model that is based on Kaganovich and Zilcha (1999), we demonstrate the introduction of education subsidies

[1] Not using the overlapping generations setting, Lucas (1988) has started to incorporate that human capital accumulation is one of the important engines of economic growth.

engenders a higher growth rate, when the governmental education tax revenue is large enough, and lead to larger growth rate, when the tax revenue is smaller because the proportion of households without children is sufficiently large.

The remainder of this paper is organized as follows. Section 2 presents the base model. Section 3 examines the growth effects of introduction of the education subsidies. Concluding remarks are presented in Section 4.

2 The Model

We consider an overlapping-generations model of endogenous growth, incorporating an uncertain lifetime. The life of a representative individual is divided into three periods: a childhood and a young-working period (each with fixed duration), and a retirement period (of uncertain length). All individuals are alive at the beginning of the third period with probability $p \in (0, 1]$. Competitive insurance companies promise individuals a payment of $(1 + \rho_{t+1}) = \frac{(1+r_{t+1})}{p}$ in exchange for having an estate a_t accruing to the companies, where $(1+r_{t+1})$ is the return of direct holdings of capital. Living individuals obtain the principal and interest from their annuities in the third period.[2]

Each individual receives education to accumulate human capital in the first period, works in the second period, and living individuals retires and receives the annuities in the third period. As we see the next subsection, only some part of households rear children in their second period. To raise a child, they are compelled to expend education investment, e_t, which helps their children to accumulate human capital. Human capital is assumed to be accumulated through private investment by parents and public schools

[2]This is a simplified version of Blanchard's (1985) model.

supplied by the government. Firms produce output using physical capital and effective labor under constant-returns-to-scale production technology. Physical capital depreciates completely after a one-period use in production. The government supplies education to children by taxing the labor income of young-working individuals. In this section, we analyze a case that the government supplies public schools, and in the nest section, we examine the effect of the introduction of education subsidies on the economic growth.

2.1 Households

The parameters of the first and the third periods are mostly common to all individuals within a given generation, except for the savings accrued. However, in the second period, individuals are divided into *two* types of households with exogenous probability. Members of the first household type are individuals who do not raise their own children. We refer to this type as Type "*A*." The second household type consists of parents who reproduce asexually and some children. We call this Type "*P*." Both type of households receive the same amount of labor income, which is taxed away.

We refer to a generation existing as workers in period t as generation t. Let N_t denote the number of working-aged individuals. We assume that the η part of workers is type "*A*," and the $(1-\eta)$ part of workers is type "*P*." Thus, the numbers of households of types "*A*" and "*P* " are given by $N_t^A = \eta N_t$ and $N_t^P = (1-\eta)N_t$, respectively. Population growth rate is written as:

$$\frac{N_{t+1}}{N_t} = \frac{n(1-\eta)N_t}{N_t} = n(1-\eta).$$

2.1.1 Type "A " Households

Each "A" household derives utility from both material consumption in the second and third periods of life. The lifetime utility function of a household in generation t is expressed as

$$u_t^A = \ln c_t^A + p\ln d_{t+1}^A, \tag{1}$$

where c_t^A and d_{t+1}^A denote consumption in the second and third periods of life.

The budget constraints of type "A" of generation t are given by

$$c_t^A + a_t^A = (1-\epsilon)w_t h_t, \tag{2}$$

$$d_{t+1}^A = (1+\rho_{t+1})a_t^A, \tag{3}$$

where a_t^A is the annuity, ϵ is the education tax rate which is constant over time, and $(1+\rho_{t+1})$ is the return of annuity.

Households choose consumption levels in the second and third periods so as to maximize lifetime utility (1) subject to the budget constraints (2) and (3). Using the first-order conditions, we obtain the optimal values of type "A" in equilibrium as:

$$c_t^A = \frac{1}{(1+p)}(1-\epsilon)w_t h_t,$$

$$d_{t+1}^A = \frac{p}{(1+p)}(1+\rho_{t+1})(1-\epsilon)w_t h_t,$$

$$a_t^A = \frac{p}{(1+p)}(1-\epsilon)w_t h_t. \tag{4}$$

2.1.2 Type "P" Households

Let h_{t+1} be the human capital level of each individual who is born at time t and called generation $t+1$. Human capital is accumulated according to:

$$h_{t+1} = (e_t)^{\gamma}(e_t^g)^{1-\gamma}. \tag{5}$$

In that equation, $\gamma \in (0,1)$ denotes the efficiency of private education, e_t, provided privately by parents, such as textbooks and tutors. The public-school quality provided by the government is e_t^g.[3] Individuals determine the amount of private investment for their children, taking the value of e_t^g as given.

Each "P" household works to receive after-tax wage income and raises n children in the second working period. She/He derives utility from human capital level of own children as well as both material consumption in the second and third periods of life.

The lifetime utility function of "P" household in generation t is expressed as[4]

$$u_t^P = \ln c_t^P + p\ln d_{t+1}^P + \delta n \ln h_{t+1}, \tag{6}$$

where c_t^P and d_{t+1}^P denote consumption in the second and third periods, $\delta(>0)$ represents the degree of preference for human capital of own children, and n is the number of children.

[3]We call e_t^g "public-school quality", as do Glomm and Ravikumar (1992), to distinguish strictly between ν_t, which is subsidized to private parental investment, and e_t^g. We can also regard e_t^g as other public educational investment like public libraries, museums, and so on.

[4]With an uncertain lifetime, $p\in(0,1]$, this utility form is employed by Yakita (2001) and Pecchenino and Pollard (2002), among others.

The budget constraints of type "P" are as follows.

$$c_t^P + ne_t + a_t^P = (1-\epsilon)w_t h_t, \tag{7}$$

and

$$d_{t+1}^P = (1+\rho_{t+1})a_t^P. \tag{8}$$

The households choose consumption in the second and third periods together with the education investment of their children so as to maximize lifetime utility (6) subject to budget constraints (5), (7) and (8).

By solving the individuals' optimization problems, the optimal values are obtained as follows:

$$e_t = \frac{\gamma\delta}{(1+p+\gamma\delta n)}(1-\epsilon)w_t h_t, \tag{9}$$

$$c_t^P = \frac{1}{(1+p+\gamma\delta n)}(1-\epsilon)w_t h_t,$$

$$d_{t+1}^P = \frac{p}{(1+p+\gamma\delta n)}(1+\rho_{t+1})(1-\epsilon)w_t h_t,$$

$$a_t^P = \frac{p}{(1+p+\gamma\delta n)}(1-\epsilon)w_t h_t. \tag{10}$$

2.2 Production

Competitive firms produce a single final good by employing both physical capital and labor input. The aggregate production function at time t is given by $Y_t = F(K_t, H_t) = K_t^{\alpha} H_t^{1-\alpha}$, where Y_t, K_t, H_t, and $\alpha \in (0,1)$ denote aggregate output, physical capital, human capital, and the share of physical capital, respectively. Human capital, H_t, is

$$H_t = h_t \eta N_t + h_t(1-\eta)N_t = h_t N_t.$$

Firms hire physical capital and effective labor inputs up to the point at which the marginal product equals the factor price:

$$(1+r_t) = \frac{\partial Y_t}{\partial K_t} = \alpha(\frac{K_t}{H_t})^{\alpha-1}$$

and

$$w_t = \frac{\partial Y_t}{\partial H_t} = (1-\alpha)(\frac{K_t}{H_t})^{\alpha}.$$

2.3 The Government

The government supplies education to children by taxing the labor income of young-working individuals.

The budget constraint of the government is

$$\epsilon w_t h_t N_t = e_t^g n(1-\eta)N_t.$$

Per-child public school investment in time t is given by

$$e_t^g = \frac{\epsilon}{n(1-\eta)} w_t h_t. \tag{11}$$

2.4 Equilibrium

We assume that individuals have perfect foresight into not only future interest rates and education subsidies but also the human capital level in the next generation.

By substituting (9) and (11) into (5), the human capital level of generation $t+1$ in the equilibrium is represented as

$$h_{t+1} = (\frac{\gamma\delta(1-\epsilon)}{(1+p+\gamma\delta n)})^{\gamma}(\frac{\epsilon}{n(1-\eta)})^{1-\gamma} w_t h_t \equiv h^* w_t h_t.$$

From this value, the capital market-clearing condition, $K_{t+1} = a_t^A \eta N_t + a_t^P(1-\eta)N_t$, and Eqs. (4) and (10), the economic growth rate in this economy at equilibrium is constant over time:

$$(1+g_t) = \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}} = (1-\alpha)\left(\frac{k^*}{n(1-\eta)}\right)^{\alpha}(h^*)^{1-\alpha} \equiv g^*, \tag{12}$$

where $k^* \equiv p(1-\epsilon)[\frac{\eta}{(1+p)} + \frac{(1-\eta)}{(1+p+\gamma\delta n)}]$.

2.5 Public Schools

Let's examine how the education tax rate affects the growth rate in the economy. The following proposition summarizes the effect of the education tax rate on the growth rate.

Proposition 1.

If $\epsilon \leq (>) \tilde{\epsilon}$, then $\text{sign}(\frac{\partial g^*}{\partial \epsilon}) \geq (<) 0$, where $\tilde{\epsilon} \equiv (1-\alpha)(1-\gamma) \in (0,1)$.

Proof. *See the Appendix A.*

This proposition shows that there is the education tax rate which maximizes the growth rate in this economy. This is the same result as the previous studies which consider homogeneous households among the generations.[5]

[5]Tournemaine and Tsoukis (2015) suppose that heterogeneity across agents is introduced via their status motivation, which affects their choice of education and shows an inverted U-shaped relationship between growth and the volume of public education.

3 Education Subsidies

In this section, we present an analysis of growth effects when the government introduces an education subsidy using a share μ of the revenue previously devoted to supply public schools in Section 2. Here, μ is treated as a predetermined parameter and is constant over time.

The value of education subsidies, ν_t, per child and public school investment, e_t^{gs}, per child at time t are adjusted to balance the governmental budget constraints. The budget constraints governing the education policy at time t are given by

Education subsidies; $\mu\epsilon w_t h_t N_t = \nu_t n(1-\eta)N_t,$

Public schools; $(1-\mu)\epsilon w_t h_t N_t = e_t^{gs} n(1-\eta)N_t.$

Thus, the value of education investment per child at time t are determined, respectively as

Education subsidies; $$\nu_t = \frac{\mu\epsilon}{n(1-\eta)} w_t h_t, \tag{13}$$

Public schools; $$e_t^{gs} = \frac{(1-\mu)\epsilon}{n(1-\eta)} w_t h_t. \tag{14}$$

3.1 Type "P" Households with Subsidies

When the government introduce the education subsidies, the budget constraint of type "P" households in the working period (7) is only rewritten as[6]

$$c_t^{Ps} + ne_t^{Ps} + a_t^{Ps} = (1-\epsilon)w_t h_t^{Ps} + n\nu_t. \tag{15}$$

From Eqs. (5), (6), (8), and (15), the optimal values are obtained by solving optimization problems to yield:

$$e_t^{Ps} = \frac{\gamma\delta}{(1+p+\gamma\delta n)}[(1-\epsilon)w_t h_t^{Ps} + n\nu_t], \tag{16}$$

$$c_t^{Ps} = \frac{1}{(1+p+\gamma\delta n)}[(1-\epsilon)w_t h_t^{Ps} + n\nu_t],$$

$$d_{t+1}^{Ps} = \frac{p}{(1+p+\gamma\delta n)}(1+\rho_{t+1})[(1-\epsilon)w_t h_t^{Ps} + n\nu_t],$$

$$a_t^{Ps} = \frac{p}{(1+p+\gamma\delta n)}[(1-\epsilon)w_t h_t^{Ps} + n\nu_t]. \tag{17}$$

3.2 Equilibrium

From Eqs. (5), (13), (14), and (16), the human capital level of generation $t+1$ in the equilibrium is represented as

$$h_{t+1}^{Ps} = (\frac{\gamma\delta\Omega}{(1+p+\gamma\delta n)})^{\gamma}(\frac{(1-\mu)\epsilon}{n(1-\eta)})^{1-\gamma} w_t h_t^{Ps} \equiv h^{**} w_t h_t^{Ps},$$

where $\Omega \equiv (1-\epsilon) + \frac{\mu\epsilon}{(1-\eta)}$.

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[6]The preference structure, and thus, the utility function and the budget constraint of old age of the type "P" are not changed after the governmental changes of the education policy. The setting of the type "A" is not changed at all.

From this value, the capital market-clearing condition, $K_{t+1} = a_t^A \eta N_t + a_t^{Ps}(1-\eta)N_t$, and Eqs. (4), (13) and (17), the economic growth rate with education subsidies at equilibrium becomes constant over time:

$$(1+g_t^{Ps}) = \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}} = (1-\alpha)(\frac{k^{**}}{n(1-\eta)})^{\alpha}(h^{**})^{1-\alpha} \equiv g^{**}, \qquad (18)$$

where $k^{**} \equiv p(1-\epsilon)[\frac{\eta}{(1+p)} + \frac{(1-\eta)}{(1+p+\gamma\delta n)}] + \frac{p\mu\epsilon}{(1+p+\gamma\delta n)} = k^* + \frac{p\mu\epsilon}{(1+p+\gamma\delta n)}$.

3.3 Equilibrium Analysis

The following proposition summarizes the effect of the education subsidies on the growth rate.

Proposition 2.

(1) If $\epsilon \geq \tilde{\epsilon}$, then $\text{sign}(\frac{\partial g^{**}}{\partial \mu}|_{\mu=0}) \geq 0$,
(2) If $\epsilon < \tilde{\epsilon}$, then $\text{sign}(\frac{\partial g^{**}}{\partial \mu}|_{\mu=0}) \geq (<) 0$, when $\eta \geq (<) \hat{\eta}$,

where $\hat{\eta} = \frac{-B+\sqrt{B^2-4AC}}{2A}$, where $A = (1-\alpha)(1-\gamma)(1-\epsilon)\frac{\gamma\delta n}{(1+p)}$, $B = -[\frac{\gamma\delta n}{(1+p)}(1-\alpha)(1-\gamma-\epsilon) - (1-\alpha)(1-\gamma)(1-\epsilon) + \alpha\epsilon]$, and $C = -\{(1-\alpha)(1-\gamma) - \epsilon\}$. $\tilde{\epsilon} \equiv (1-\tau)(1-\gamma) \in (0,1)$ is the same value of Proposition 1.

Proof. *See the Appendix B.*

This proposition shows that the growth effect of child subsidy depends on the payroll tax rate, ϵ, and the proportion of households without children, η. When the tax rate and thereby the amount of education revenue is large enough ($\epsilon \geq \tilde{\epsilon}$), the introduction of child subsidy increases the growth rate (case 1). In the contrary case (case 2), when the tax rate is small enough

($\epsilon < \tilde{\epsilon}$), only the economy in which the proportion of households without children is large enough ($\eta \geq \hat{\eta}$), the growth increases by the introduction of child subsidy because the amount of tax revenue is large as compared with smaller number of the recipients.

Although the threshold levels of parental preferences for their children hold important meaning for results revealed in Kaganovich and Zilcha (1999), the levels of preferences vary among individuals and are difficult to measure. Our contribution is to have demonstrated that two observable parameters – the proportion of household that do not raising children, η, and the education-tax rates, ϵ,– become thresholds of determining the subsidies' effects by incorporating the differences of household in the point of having children into the model of Kaganovich and Zilcha (1999).

4 Concluding Remarks

We have examined how the volume of public schools and the introduction of education subsidies affect growth rates, incorporating an uncertain lifetime and family configuration. We show that there is the education tax rate which maximizes the growth rate which is the same result as the previous researches which consider homogeneous households among the generations. We also demonstrate that introducing subsidies engenders higher growth rates depending on the education tax rate and the proportion of households that do not raise children in the economy.

Appendix A

Proof of Proposition 1.

By differentiating (12) with respect ϵ, we obtain

$$sign(\frac{\partial g^*}{\partial \epsilon}) \gtrless 0$$

$$\iff sign(\frac{\partial((1-\epsilon)^{\alpha+\gamma(1-\alpha)}\epsilon^{(1-\alpha)(1-\gamma)})}{\partial \epsilon}) \gtrless 0$$

$$\iff sign(-\{\alpha+\gamma(1-\alpha)\}\epsilon+(1-\alpha)(1-\gamma)(1-\epsilon)) \gtrless 0$$

$$\iff sign(-\epsilon+(1-\alpha)(1-\gamma) \gtrless 0$$

$$\iff \epsilon \lesseqgtr (1-\alpha)(1-\gamma) \equiv \tilde{\epsilon}.$$

This is the proof of Proposition 1.

Q.E.D.

Appendix B

Proof of Proposition 2.

By differentiating (18) with respect μ and valuing it at $\mu = 0$, we obtain the following equation:

$$sign(\frac{\partial g^{**}}{\partial \mu}|_{\mu=0})$$

$$= sign((1-\alpha)(1-\gamma)(1-\epsilon)\frac{\gamma\delta n}{(1+p)}\eta^2$$

$$-[\frac{\gamma\delta n}{(1+p)}(1-\alpha)(1-\gamma-\epsilon)-(1-\alpha)(1-\gamma)(1-\epsilon)+\alpha\epsilon]\eta$$

$$-\{(1-\alpha)(1-\gamma)-\epsilon\}) \equiv signG(\eta).$$

$G(\eta)$ is a quadratic function of η. The graph of it is convex downward. By the assumption of $\eta \in (0,1)$ and

$$signG(\eta = 1) = \gamma\epsilon[\frac{\gamma\delta n}{(1+p)}(1-\alpha) + \gamma\epsilon] > 0,$$

when $signG(\eta = 0) \geq 0$, that is,

$$signG(\eta = 0) = -\{(1-\alpha)(1-\gamma) - \epsilon\} \geq 0$$

$$\Longleftrightarrow \epsilon \geq (1-\alpha)(1-\gamma) \equiv \tilde{\epsilon},$$

it becomes $signG(\eta) > 0$ for all $\eta \in (0,1)$.[7]

When $signG(\eta = 0) < 0$, that is $\epsilon < \tilde{\epsilon}$, it exists $\hat{\eta}$ which intersects the x-axis once in $\eta \in (0,1)$. As the intersection is a larger solution of $G(\eta) = 0$, we get the value as

$$\hat{\eta} = \frac{-B + \sqrt{B^2 - 4AC}}{2A},$$

where $A = (1-\alpha)(1-\gamma)(1-\epsilon)\frac{\gamma\delta n}{(1+p)}$, $B = -[\frac{\gamma\delta n}{(1+p)}(1-\alpha)(1-\gamma-\epsilon) - (1-\alpha)(1-\gamma)(1-\epsilon) + \alpha\epsilon]$, and $C = -\{(1-\alpha)(1-\gamma) - \epsilon\}$.

This is the proof of Proposition 2.

Q.E.D.

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[7] To avoid the complication, we assume that

$$[\frac{\gamma\delta n}{(1+p)}(1-\alpha)(1-\gamma-\epsilon) - (1-\alpha)(1-\gamma)(1-\epsilon) + \alpha\epsilon] > 0$$

to ensure that the function has only one intersect in x-axis in $\eta \in (0,1)$

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